# 2024 F=ma Exam: Problem 23

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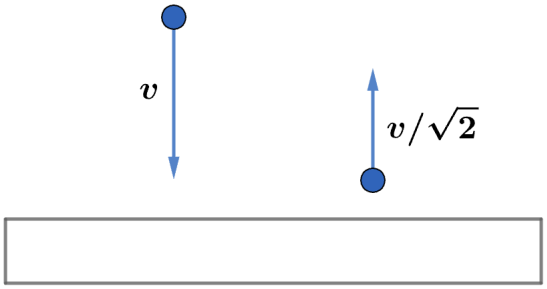
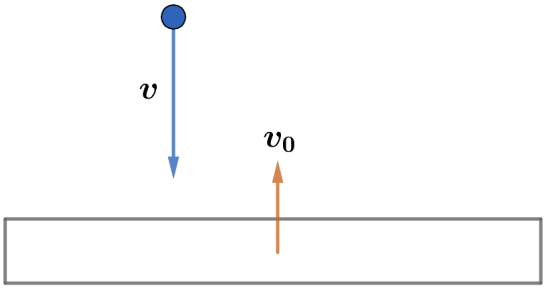
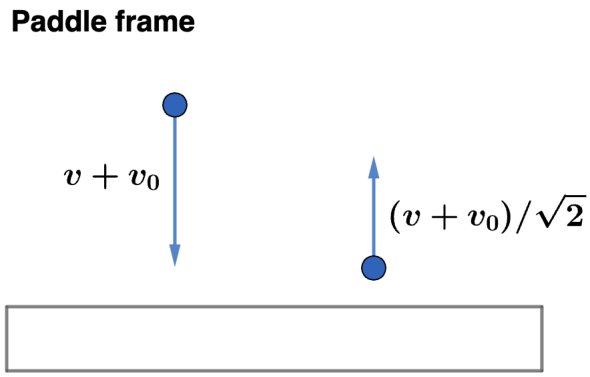
Since the ball bounces back up to half the initial height $h$, it loses half its energy $E$ after colliding with a stationary paddle. We have $E \propto v^2$ so its speed is reduced by a factor of $\sqrt{2}$.



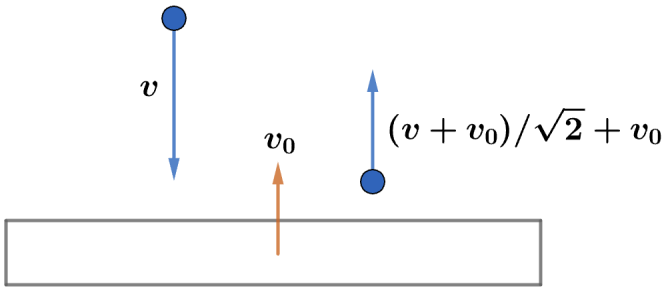

Now we let the paddle move such that it's moving upward with speed $v_0$ when it collides with the ball.




To determine what happens, we can go to the frame of the paddle where we know the ball's speed gets reduced by a factor of $\sqrt{2}$:




We go back to the ground frame by adding velocity $v_0$ upward on all objects:




Thus, after colliding with the moving paddle, the ball has speed

$$v_f = \frac{v + v_0}{\sqrt{2}} + v_0$$

If the ball returns to the same height, then we must have

$$v = v_f = \frac{1}{\sqrt{2}} v + \left(1 + \frac{1}{\sqrt{2}}\right) v_0$$

$$v = \frac{1 + (1/\sqrt{2})}{1 - (1/\sqrt{2})} v_0 = \frac{\sqrt{2} + 1}{\sqrt{2} - 1} v_0$$

By conservation of energy,

$$mgh = \frac{1}{2} m v^2$$

$$h = \frac{v^2}{2g} = \left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1}\right)^2 \frac{v_0^2}{2g} = 1.7\,\text{m}$$

so the answer is $\boxed{\text{D}}$.