2024 F=ma Exam: Problem 20

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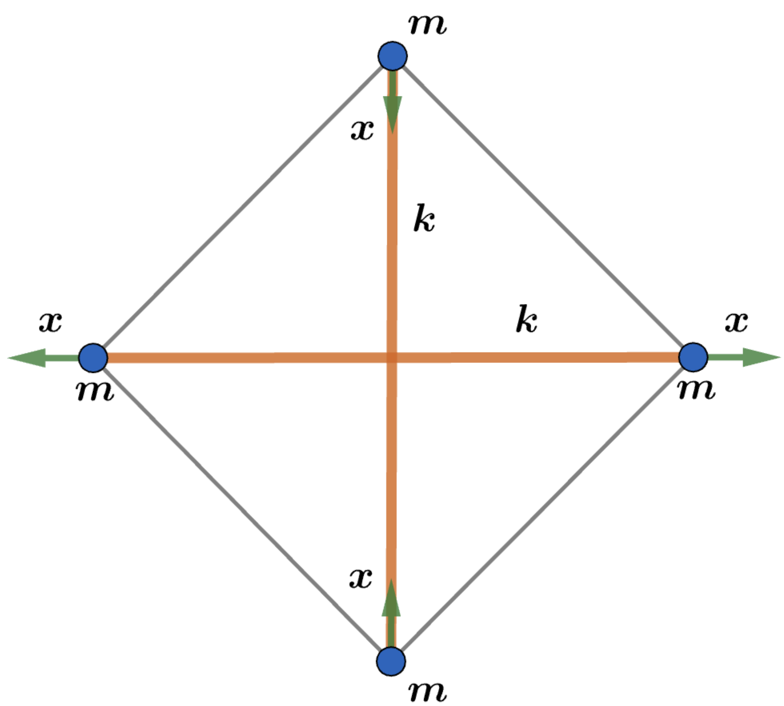

Suppose we compress along one diagonal by displacing the top and bottom masses each by x towards each other. By conservation of rod length, the two masses in the middle move apart, each displacing by x also. We derive the equation of motion for $x(t)$ by energy conservation. The kinetic energy of the system is given by

$$K = 4\left(\frac{1}{2}m\dot{x}^2\right)$$

since all masses move by the same amount and have the same speed. The potential energy of the system is given by

$$U = 2\left[\frac{1}{2}k(2x)^2\right]$$

since both springs change length by $2x$. We have

$$E = K + U = 2m\dot{x}^2 + 4kx^2$$

Conserving energy,

$$0 = \frac{dE}{dt} = 2m(2\dot{x}\ddot{x}) + 4k(2x\dot{x}) = \dot{x}(4m\ddot{x} + 8kx)$$

$$m\ddot{x} + 2kx = 0$$
$$\ddot{x} = -\frac{2k}{m}x$$

which is of simple harmonic form $\ddot{x} = -\omega^2 x$ with

$$\omega = \sqrt{\frac{2k}{m}}$$

Thus the period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}$$

so the answer is $\boxed{\text{B}}$.