2024 F=ma Exam: Problem 19

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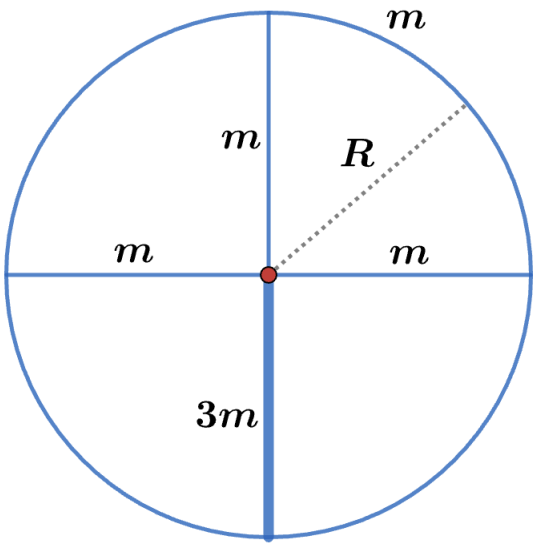


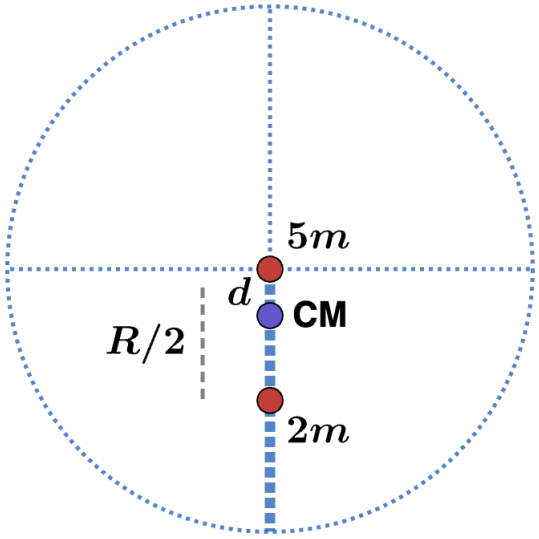
Recall the angular frequency of a physical pendulum is given by

$$\omega = \sqrt{\frac{Mgd}{I}}$$

where M is the total mass, d is the distance between the CM and pivot point, and I is the total moment of inertia. We have

$$M = m + m + m + m + 3m = 7m$$
$$I = mR^2 + 3\left(\frac{1}{3}mR^2\right) + \frac{1}{3}(3m)R^2 = 3mR^2$$

To find the CM, we can think of the wheel as the superposition of a spoke of mass $2m$ with a symmetric wheel that has four spokes of equal mass m. The CM of each object is at its center so we can place all their mass at those points:



Then the CM of the entire system is at

$$d = \frac{2m(R/2)}{2m + 5m} = \frac{R}{7}$$

Thus,

$$\omega = \sqrt{\frac{(7m)g(R/7)}{3mR^2}} = \sqrt{\frac{g}{3R}}$$

so the answer is $\boxed{\text{A}}$.