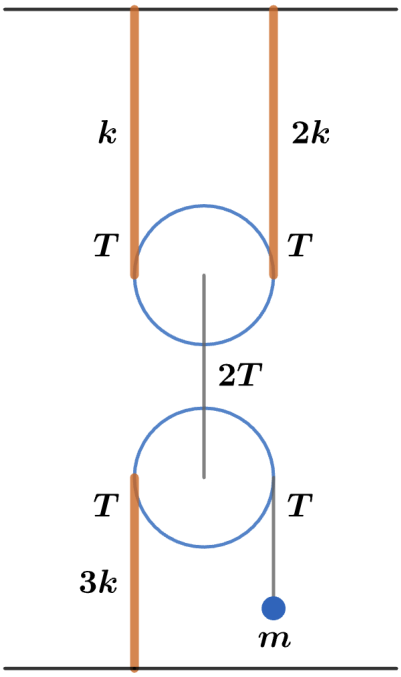
2024 F=ma Exam: Problem 17

Kevin S. Huang



Let T be the tension in the bottom string (attached to the mass m). Then balancing forces on the pulleys, we find the tensions in the springs are also T. Looking at the mass, we have

$$T = mg$$

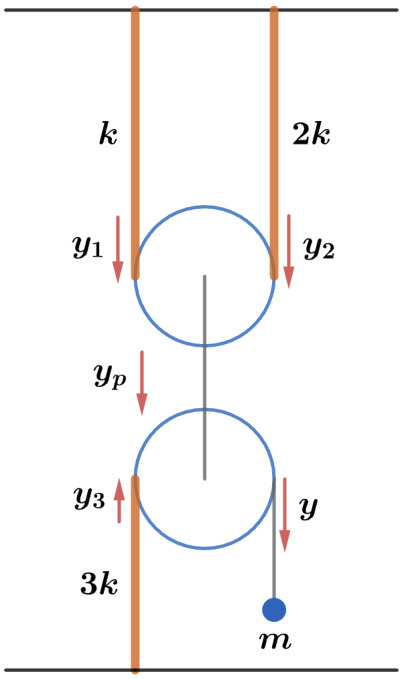
After the mass is doubled,

$$T' = 2mg$$

and the tension in the springs increases by

$$\Delta T = T' - T = mg$$

so each spring extends by $y_i = mg/k_i$. We use conservation of string to find the displacement y of the mass.



If the pulleys move down by y_p (they move together to keep the middle string's length fixed) then the total length of the top two springs increases by $2y_p$. This has to be equal to their actual increase in length of $y_1 + y_2$,

$$2y_p = y_1 + y_2$$

If the pulleys move down by y_p then the length of the bottom *string* decreases by $2y_p$. Since the bottom spring increases in length by y_3, the bottom *string* decreases in length by y_3. The mass moves down by y, so the string increases in length by y. Conserving string length,

$$y = 2y_p + y_3$$

Solving for y,

$$y = y_1 + y_2 + y_3 = \frac{mg}{k} + \frac{mg}{2k} + \frac{mg}{3k} = \frac{11}{6}\frac{mg}{k}$$

so the answer is $\boxed{\text{E}}$.