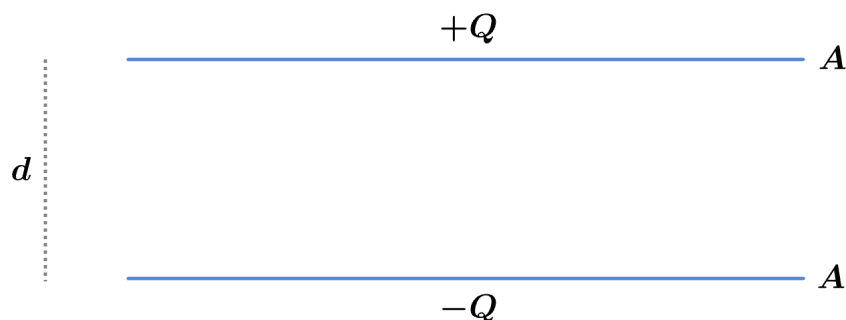


## 2009 Quarter-final Exam: Problem 3

Kevin S. Huang

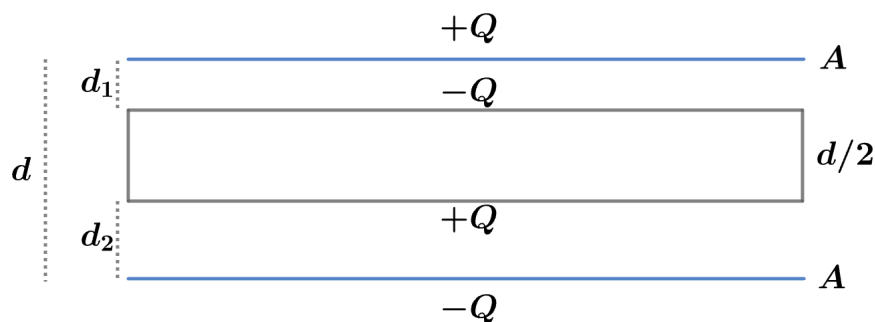


Once the capacitor is fully charged to charge  $Q = CV$ , the energy stored is

$$U_i = \frac{Q^2}{2C}$$

Recall for a parallel plate capacitor,  $C = \epsilon_0 A/d$  so

$$U_i = \frac{Q^2 d}{2\epsilon_0 A}$$



After the conducting slab is inserted, surface charges  $\pm Q$  are induced such that there is no electric field in the conductor. We effectively have two capacitors connected in parallel. Using our earlier formula, the total energy stored is now

$$U_f = U_1 + U_2 = \frac{Q^2 d_1}{2\epsilon_0 A} + \frac{Q^2 d_2}{2\epsilon_0 A} = \frac{Q^2 d}{4\epsilon_0 A}$$

since  $d_1 + d_2 = d - d/2 = d/2$ . The work done is equal to the change in energy,

$$W = \Delta U = U_f - U_i = \frac{Q^2 d}{4\epsilon_0 A} - \frac{Q^2 d}{2\epsilon_0 A} = -\frac{Q^2 d}{4\epsilon_0 A}$$

Plugging in  $Q = CV = \epsilon_0 AV/d$ ,

$$W = -\left(\frac{\epsilon_0 AV}{d}\right)^2 \frac{d}{4\epsilon_0 A} = -\frac{\epsilon_0 AV^2}{4d}$$