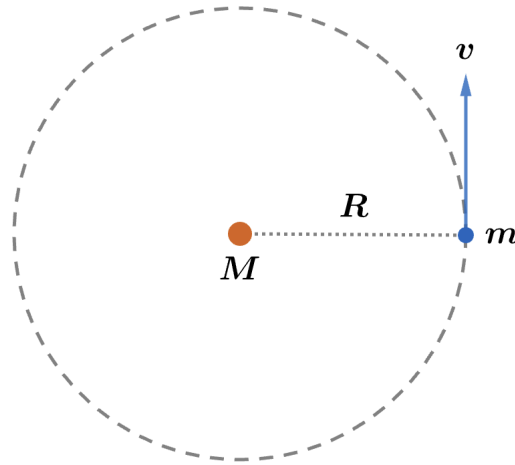


2009 Quarter-final Exam: Problem 1

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a) Since the mass m of the planet is much less than the mass M of the star, we can assume the star has a fixed position. Recall the velocity v of a circular orbit of radius R is given by

$$v = \sqrt{\frac{GM}{R}}$$

Because the orbit has circumference $2\pi R$, the period T of the orbit is

$$T = \frac{2\pi R}{v} = 2\pi \sqrt{\frac{R^3}{GM}}$$

Solving for M ,

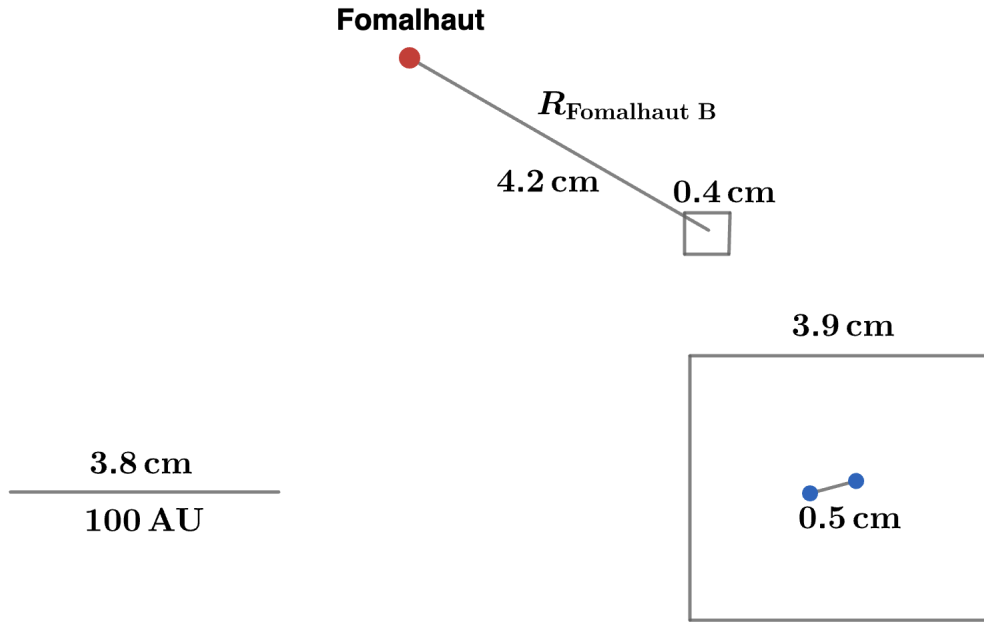
$$\left(\frac{T}{2\pi}\right)^2 = \frac{R^3}{GM}$$

$$M = \frac{4\pi^2 R^3}{GT^2}$$

b) Since $M \propto R^3/T^2$, we have

$$\frac{M_{\text{Fomalhaut}}}{M_{\text{sun}}} = \frac{R_{\text{Fomalhaut B}}^3 T_{\text{Earth}}^2}{T_{\text{Fomalhaut B}}^2 R_{\text{Earth}}^3} = \frac{(R_{\text{Fomalhaut B}}/1 \text{ AU})^3}{(T_{\text{Fomalhaut B}}/1 \text{ yr})^2}$$

Measuring the various distances in the diagram, we obtain:



Because 3.8 cm represents 100 cm, we have

$$R_{\text{Fomalhaut B}} = 4.2 \text{ cm} \left(\frac{100 \text{ AU}}{3.8 \text{ cm}} \right) = 110 \text{ AU}$$

The inset is a factor of $3.9 \text{ cm}/0.4 \text{ cm} = 9.7$ larger than the diagram, so Fomalhaut B moves a distance of

$$d = \frac{0.5 \text{ cm}}{9.7} \left(\frac{100 \text{ AU}}{3.8 \text{ cm}} \right) = 1.4 \text{ AU}$$

over $t = 2006 - 2004 = 2 \text{ yrs}$. Since $v = d/t$, we have

$$T_{\text{Fomalhaut B}} = \frac{2\pi R_{\text{Fomalhaut B}}}{v} = 2\pi R_{\text{Fomalhaut B}} \left(\frac{t}{d} \right) = 2\pi(110 \text{ AU}) \left(\frac{2 \text{ yrs}}{1.4 \text{ AU}} \right) = 980 \text{ yrs}$$

Finally,

$$\boxed{\frac{M_{\text{Fomalhaut}}}{M_{\text{sun}}} = \frac{110^3}{980^2} = 1.4}$$