2008 Quarter-final Exam: Problem 1

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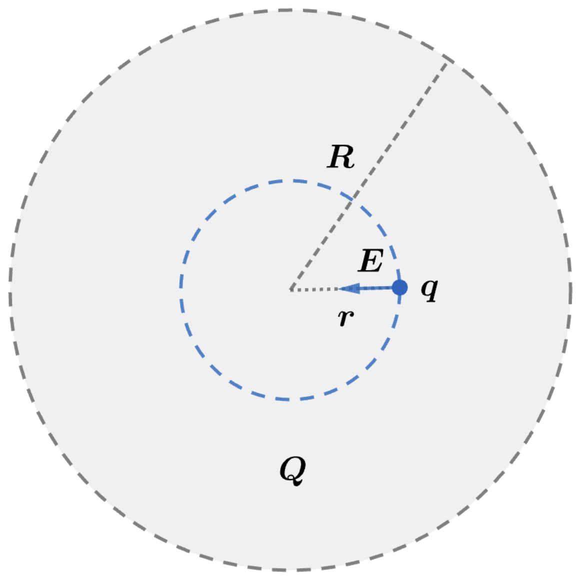
We can apply Gauss's law to find the force on charge q when it is distance r from the center of the sphere.



We have

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q(r^3/R^3)}{\epsilon_0}$$

using the fact that for uniform charge density, $q_{\text{enc}} \propto r^3$. The electric field and force on q is then

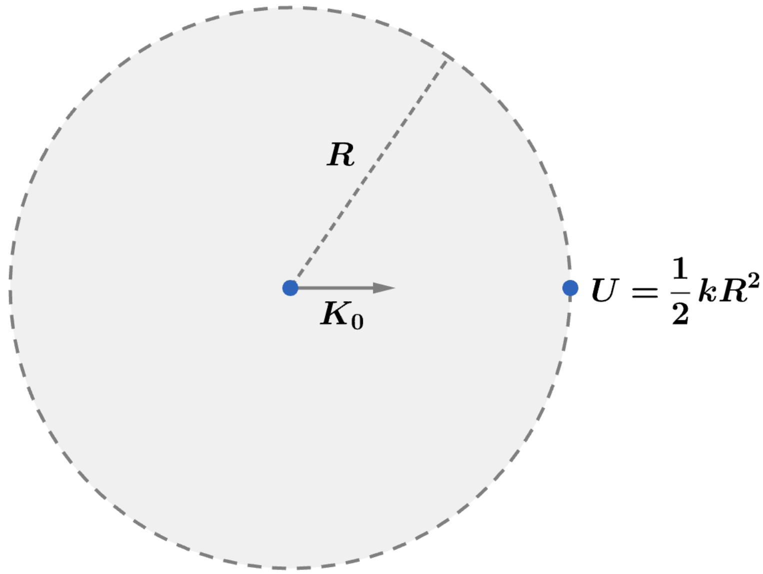
$$E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}$$

$$F = qE = \frac{1}{4\pi\epsilon_0}\frac{qQ}{R^3}r$$

Since $qQ < 0$, the particle undergoes simple harmonic motion with restoring force $F = -kr$ where we can identify

$$k \equiv -\frac{1}{4\pi\epsilon_0}\frac{qQ}{R^3}$$

a) Recall the potential energy of a simple harmonic oscillator is $U = \frac{1}{2}kx^2$.



The kinetic energy of the particle is all converted to potential energy when it reaches the boundary,

$$K_0 = \frac{1}{2}kR^2$$

$$\boxed{K_0 = -\frac{1}{8\pi\epsilon_0}\frac{qQ}{R}}$$

b) Recall the period of a simple harmonic oscillator is $T = 2\pi\sqrt{\frac{m}{k}}$. Moving from the equilibrium to the amplitude takes a quarter of the period so

$$t_q = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{m}{k}}$$

$$\boxed{t_q = \frac{\pi}{2}\sqrt{\frac{4\pi\epsilon_0 mR^3}{-qQ}}}$$