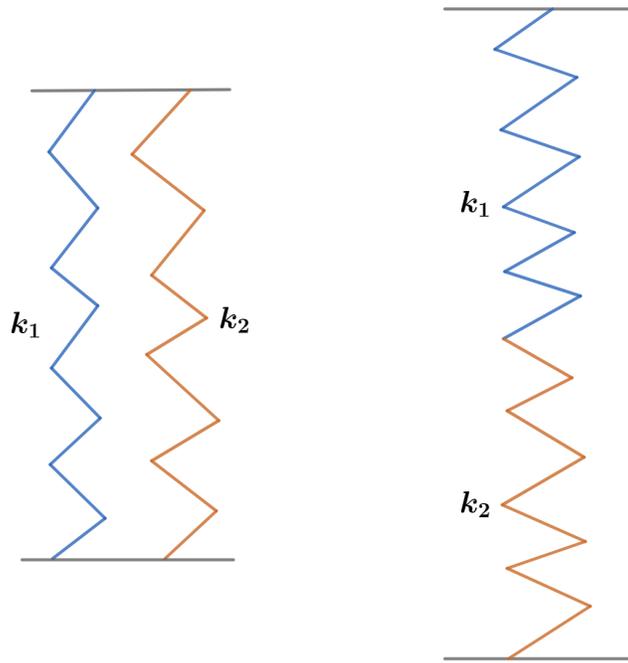


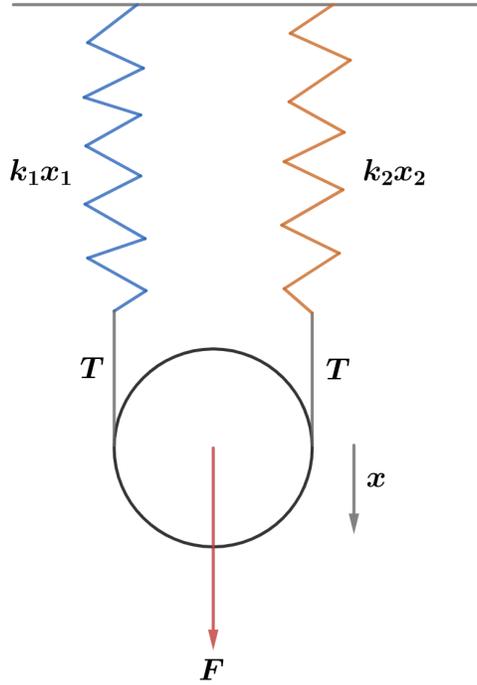
2023 F=ma Exam: Problem 22

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Recall spring constants add for springs in parallel and the reciprocals add for springs in series. Thus,

$$\begin{aligned}k_A &= k_1 + k_2 \\ \frac{1}{k_C} &= \frac{1}{k_1} + \frac{1}{k_2} \\ k_C &= \frac{k_1 k_2}{k_1 + k_2}\end{aligned}$$



In the second case, if we apply a force F and displace the pulley by x , then the effective spring constant is $k_B = F/x$.

The first spring extends by x_1 and the second spring extends by x_2 . By conservation of string,

$$x_1 + x_2 = 2x$$

Balancing forces on the pulley,

$$F = 2T$$

while balancing forces on the string,

$$T = k_1 x_1 = k_2 x_2$$

Thus, we have

$$\begin{aligned} k_B &= \frac{F}{x} = \frac{2T}{(x_1 + x_2)/2} = \frac{4k_1 x_1}{x_1 + x_2} \\ &= \frac{4k_1}{1 + (x_2/x_1)} = \frac{4k_1}{1 + (k_1/k_2)} = \frac{4k_1 k_2}{k_1 + k_2} \end{aligned}$$

Note that $k_A > k_B$ since

$$\begin{aligned} k_1 + k_2 &> \frac{4k_1 k_2}{k_1 + k_2} \\ (k_1 + k_2)^2 &> 4k_1 k_2 \\ k_1^2 - 2k_1 k_2 + k_2^2 &> 0 \\ (k_1 - k_2)^2 &> 0 \end{aligned}$$

as required. Hence, $k_A > k_B > k_C$ so the answer is A.