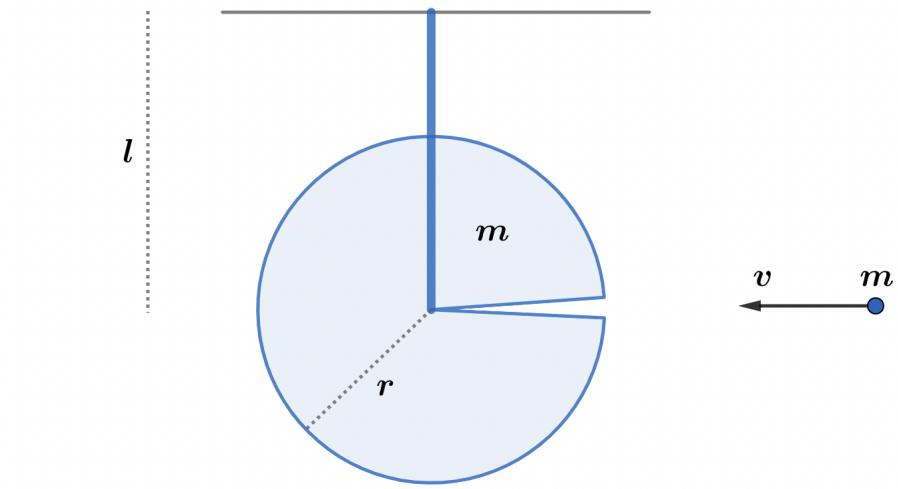
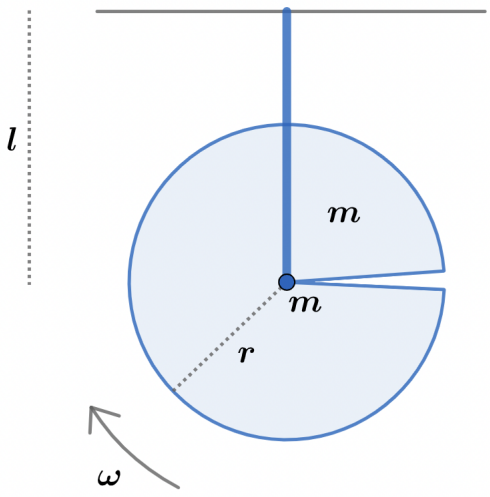
2022B F=ma Exam: Problem 13

Kevin S. Huang



The initial angular momentum about the pivot is

$$L_i = mvl$$



The final angular momentum after the collision is

$$L_f = I\omega$$

where

$$I = I_{\text{ball}} + I_{\text{block}} = \left(\frac{2}{5}mr^2 + ml^2\right) + ml^2 = 2m\left(\frac{r^2}{5} + l^2\right)$$

The kinetic energy of the system after the collision is

$$K = \frac{L_f^2}{2I} = \frac{L_i^2}{2I} = \frac{m^2v^2l^2}{4m\left(\frac{r^2}{5} + l^2\right)} = \frac{mv^2}{4\left(1 + \frac{r^2}{5l^2}\right)}$$

using the fact that angular momentum is conserved. Conserving energy when the pendulum swings up,

$$K = U_g$$

$$\frac{mv^2}{4\left(1 + \frac{r^2}{5l^2}\right)} = (2m)gh$$

$$h = \frac{v^2}{8g\left(1 + \frac{r^2}{5l^2}\right)}$$

so the answer is $\boxed{\text{D}}$.