2021 F=ma Exam: Problem 25

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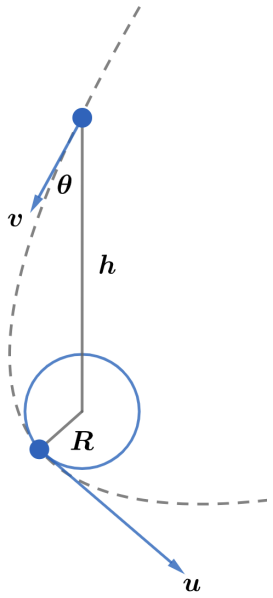

Since a pellet launched directly away from Orb can just barely escape, we have

$$v = \sqrt{\frac{2GM}{h}}$$

using the expression for the escape velocity. If the pellet's minimum distance to Orb is less than R then it lands on the planet. Consider a pellet launched at angle θ_c from the vertical such that its minimum distance to Orb is R. Conserving angular momentum about the planet, we have

$$mvh\sin\theta_c = muR$$

Since the energy of the orbit is zero, we have $u = \sqrt{2GM/R}$. Simplifying,

$$m\sqrt{\frac{2GM}{h}}h\sin\theta_c = m\sqrt{\frac{2GM}{R}}R$$
$$\sqrt{h}\sin\theta_c = \sqrt{R}$$
$$\sin\theta_c \approx \theta_c = \sqrt{\frac{R}{h}}$$

For $|\theta| < \theta_c$, the pellet collides with the planet. The fraction is

$$f = \frac{\theta_c - (-\theta_c)}{2\pi} = \frac{1}{\pi}\sqrt{\frac{R}{h}}$$

so the answer is $\boxed{\text{E}}$.