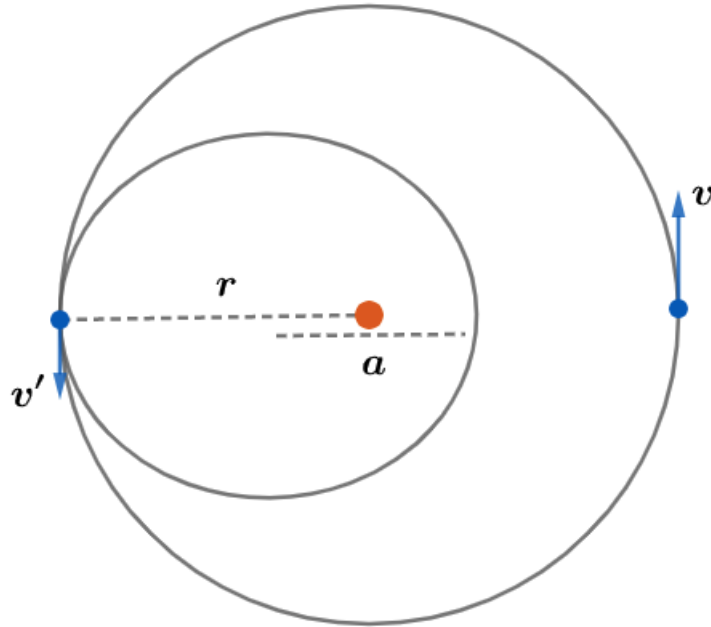


# 2021 F=ma Exam: Problem 24

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To meet the second satellite in time  $T/2$ , the first satellite should have new period  $T' = T/2$ . By Kepler's 3rd law,

$$\begin{aligned}\frac{T'^2}{T^2} &= \frac{a^3}{r^3} \\ a^3 &= r^3/4 \\ a &= \frac{r}{4^{1/3}}\end{aligned}$$

Recall the energy of an orbit is given by

$$E = -\frac{GMm}{2a}$$

so the first satellite should adjust its velocity to

$$\begin{aligned}\frac{1}{2}mv'^2 - \frac{GMm}{r} &= -\frac{GMm}{2a} \\ \frac{1}{2}mv'^2 &= \frac{GMm}{r} - \frac{GMm}{2^{1/3}r}\end{aligned}$$

$$v' = \sqrt{\frac{2GM}{r} \left(1 - \frac{1}{2^{1/3}}\right)}$$

For a circular orbit, recall

$$v = \sqrt{\frac{GM}{r}}$$

so

$$v' = \sqrt{2 - 2^{2/3}}v = 0.64v$$

Thus, the answer is B.