2020B F=ma Exam: Problem 17

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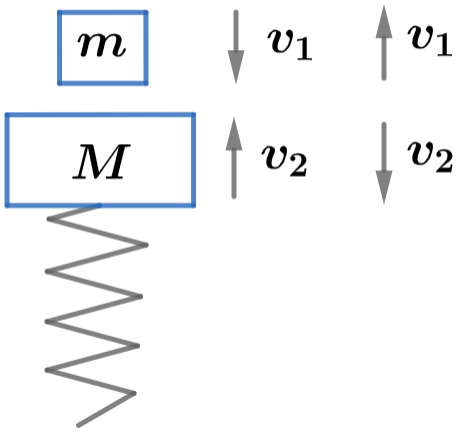

For m to rebound to its original height, we know its velocity flips after the elastic collision. Since energy is conserved, we know M has the same speed as before. Furthermore, from conservation of momentum, its velocity also flips. Thus,

$$p = mv_1 - Mv_2 = 0$$

From conservation of energy for m,

$$mgh = \frac{1}{2}mv_1^2$$
$$v_1 = \sqrt{2gh}$$

From conservation of energy for M,

$$\frac{1}{2}k\Delta x^2 = \frac{1}{2}Mv_2^2$$
$$v_2 = \Delta x\sqrt{\frac{k}{M}}$$

Substituting into the earlier equation,

$$m\sqrt{2gh} = M\Delta x\sqrt{\frac{k}{M}}$$
$$\Delta x = \frac{m}{M}\sqrt{\frac{2Mgh}{k}}$$

From the last problem,

$$h = \frac{\pi^2 Mg}{8k}$$

Thus,

$$\Delta x = \frac{m}{M}\sqrt{\frac{2\pi^2 M^2 g^2}{8k^2}} = \frac{m}{M}\frac{\pi Mg}{2k} = \frac{\pi mg}{2k}$$

so the answer is $\boxed{\text{E}}$.