2018A F=ma Exam: Problem 4

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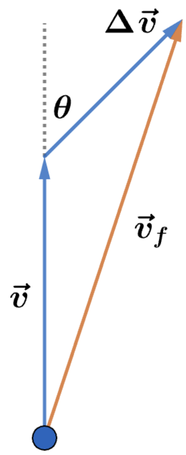

The initial energy of the satellite is given by

$$E_0 = \frac{1}{2}m\vec{v}^{\,2} - \frac{GM_E m}{r}$$

By the impulse-momentum theorem, the velocity of the satellite will change by a fixed amount.

$$\vec{J} = m\Delta\vec{v}$$

The final energy of the satellite is given by

$$E_f = \frac{1}{2}m(\vec{v} + \Delta\vec{v})^2 - \frac{GM_e m}{r}$$

We want to maximize $\Delta E = E_f - E_i$:

$$\begin{aligned}
\Delta E &= \left[\frac{1}{2}m(\vec{v} + \Delta\vec{v})^2 - \frac{GM_E m}{r}\right] - \left[\frac{1}{2}m\vec{v}^2 - \frac{GM_e m}{r}\right] \\
&= \frac{1}{2}m(\vec{v}^{\,2} + 2\vec{v}\cdot\Delta\vec{v} + \Delta\vec{v}^{\,2}) - \frac{1}{2}m\vec{v}^{\,2} \\
&= m\left(\vec{v}\cdot\Delta\vec{v} + \frac{\Delta\vec{v}^2}{2}\right)
\end{aligned}$$

$$\Delta E = m\left(|v||\Delta v|\cos\theta + \frac{|\Delta v|^2}{2}\right)$$

Since $|\Delta v|$ is fixed, we want to maximize $|v|$ and $\cos\theta$. Therefore, the impulse should be directed along the satellite's velocity ($\theta = 0$ so $\cos\theta = 1$) and at the perigee ($|v|$ is largest) so the answer is $\boxed{\text{A}}$.