2018A F=ma Exam: Problem 23

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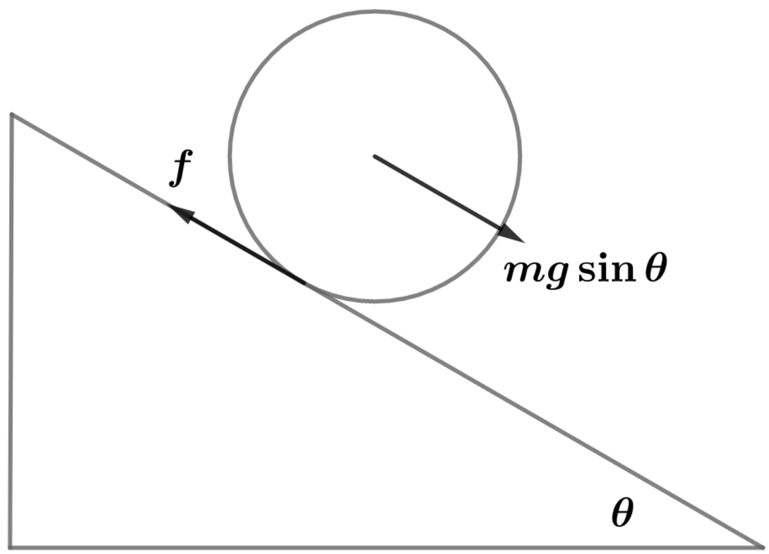

For rolling without slipping, we have $a = R\alpha$ where

$$mg\sin\theta - f = ma \tag{1}$$

$$fR = I\alpha = \beta m R^2 \alpha \tag{2}$$

Multiplying equation (1) by R and adding to equation (2) gives

$$mgR\sin\theta = \beta m R^2\alpha + mRa$$

$$mgR\sin\theta = (1+\beta)mR^2\alpha$$

$$\alpha = \frac{g\sin\theta}{(1+\beta)R}$$

Once the ball starts slipping,

$$f = \mu m g\cos\theta$$

so from equation (2),

$$\mu m g R\cos\theta = \beta m R^2\alpha$$

$$\alpha = \frac{\mu g\cos\theta}{\beta R}$$

The critical angle for the transition is

$$\frac{g\sin\theta_c}{(1+\beta)R} = \frac{\mu g\cos\theta_c}{\beta R}$$

$$\tan\theta_c = \frac{\mu(1+\beta)}{\beta}$$

Thus,

$$\alpha = \begin{cases} \frac{g\sin\theta}{(1+\beta)R} & 0<\theta<\theta_c \\ \frac{\mu g\cos\theta}{\beta R} & \theta_c<\theta<\pi/2 \end{cases}$$

so the answer is $\boxed{\text{C}}$.