2017 F=ma Exam: Problem 5

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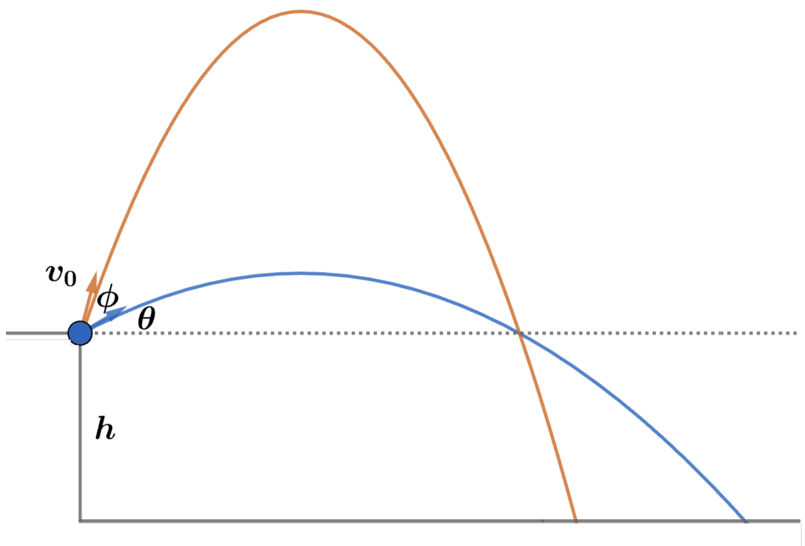
Since the optimal angle θ_{opt} should depend on the height h of the cliff, we know it should not take only one value e.g. 0 or $\pi/4$. It remains to determine whether $0 < \theta_{\text{opt}} < \pi/4$ or $\pi/4 < \theta_{\text{opt}} < \pi/2$.



Recall the range equation,

$$R = \frac{v^2 \sin(2\theta)}{g}$$

Note that for any R that can be reached, there are two corresponding launch angles θ, ϕ that get to that range where $\phi = \frac{\pi}{2} - \theta$ since

$$\sin(2\phi) = \sin(\pi - 2\theta) = \sin(2\theta)$$

Without loss of generality, let $\theta < \pi/4$ so $\phi > \pi/4$. Then we reach the same place when the projectile falls back to the starting height whether we launch with angle θ or ϕ. However, at this point, the horizontal (vertical) component of the velocity is larger (smaller) for θ compared to ϕ. Thus, the projectile launched at angle θ travels further in the horizontal direction by the time it reaches the ground.

We see that for any angle $\phi > \pi/4$, its complementary angle results in a larger horizontal range. Thus, we must have $0 < \theta_{\text{opt}} < \pi/4$ so the answer is $\boxed{\text{C}}$.