2017 F=ma Exam: Problem 24

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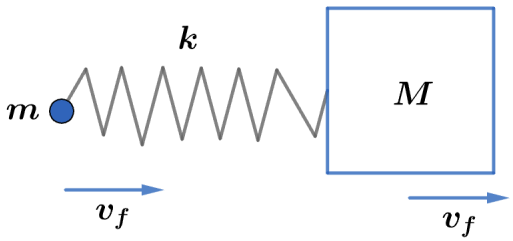
Initially, we have linear momentum

$$p = mv$$

and energy

$$E = \frac{1}{2}mv^2$$

Once mass m reaches the spring, it starts getting compressed because there is a relative velocity between m and M. Maximum compression occurs when the relative velocity goes to zero i.e. both masses move at the same velocity v_f.



The linear momentum is now

$$p = (m + M)v_f$$

and the energy

$$E = K + U = \frac{1}{2}(m + M)v_f^2 + \frac{1}{2}kx^2$$

Conserving linear momentum,

$$mv = (m + M)v_f$$

$$v_f = \frac{mv}{m + M}$$

Conserving energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2 + \frac{1}{2}kx^2$$

Solving for x,

$$mv^2 = \frac{m^2v^2}{m+M} + kx^2$$

$$kx^2 = mv^2\left(1 - \frac{m}{m+M}\right) = \frac{mMv^2}{m+M}$$

$$x = v\sqrt{\frac{mM}{k(m+M)}}$$

so the answer is $\boxed{\text{A}}$.