2017 F=ma Exam: Problem 23

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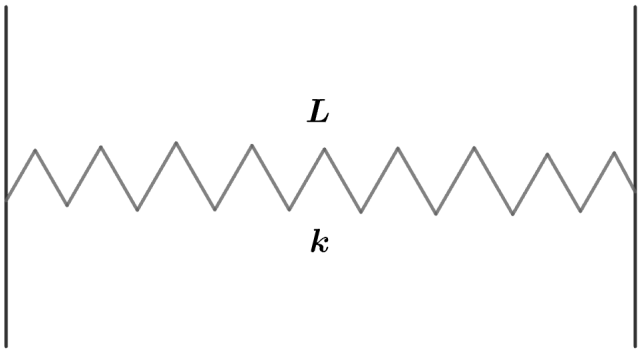

Recall the velocity of a wave pulse in a string is given by

$$v = \sqrt{\frac{T}{M/L}}$$

where T is the tension, M is the mass, and L is the length of the string. In our case for a spring, we have

$$T = k\Delta L = k(L - L_0)$$

where L_0 is the initial length of the spring. Then

$$v = \sqrt{\frac{kL(L - L_0)}{M}}$$

The time for a wave pulse to travel across the length of the spring is

$$t = \frac{L}{v} = L\sqrt{\frac{M}{kL(L - L_0)}} \propto \sqrt{\frac{L}{L - L_0}}$$

Comparing times when the spring is stretched to two different lengths, we have

$$\frac{t_2}{t_1} = \sqrt{\frac{L_2}{L_2 - L_0}}\sqrt{\frac{L_1 - L_0}{L_1}}$$

We are given $t_1 = 1\,\text{s}$, $L_0 = 1\,\text{m}$, $L_1 = 10\,\text{m}$, and $L_2 = 20\,\text{m}$. Solving for t_2,

$$t_2 = (1\,\text{s})\sqrt{\frac{20}{20 - 1}}\sqrt{\frac{10 - 1}{10}} \approx 1\,\text{s}$$

so the answer is $\boxed{\text{C}}$.