2017 F=ma Exam: Problem 11

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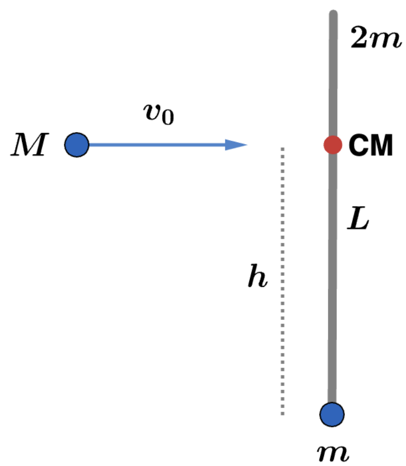


The final angular momentum (after mass M sticks to the rod-sphere object) of the system is

$$L_f = L_{\mathrm{CM}} + L_{\mathrm{relative}} = (M + 2m + m)v_{\mathrm{CM}}r + I_{\mathrm{CM}}\omega$$

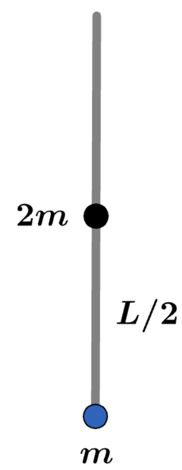
where r is the moment arm of the CM to the axis of rotation and ω is the angular velocity about the CM. If we choose the axis of rotation to be at the CM, then $r = 0$ by definition:

$$L_f = I_{\mathrm{CM}}\omega$$

For no rotation $\omega = 0$, we must have $L_f = 0$. By conservation of angular momentum,

$$0 = L_f = L_i = Mv_0r_0$$

where r_0 is the moment arm (relative to the CM) of mass M. Since $r_0 = 0$, the mass should hit the CM of the rod-sphere object.



To find the CM of the rod-sphere object, we can replace the rod with a point mass $2m$ at its center. Then, the CM is located distance

$$h = \frac{2m(L/2)}{m + 2m} = \frac{L}{3}$$

from the bottom. Thus, the answer is $\boxed{\text{B}}$.