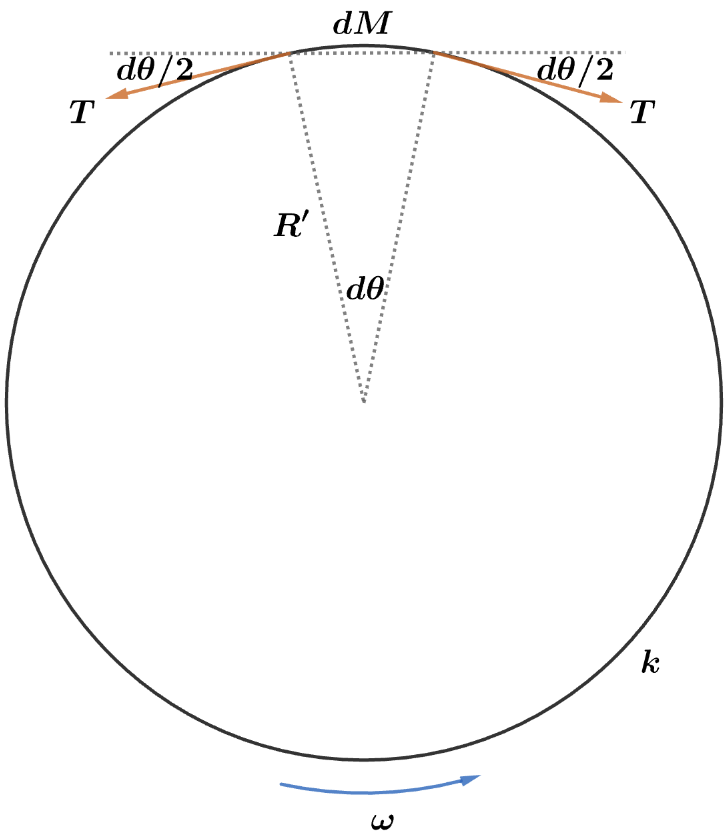
2016 F=ma Exam: Problem 23

Kevin S. Huang



We analyze a small piece of the rubber band with mass dM that subtends an angle $d\theta$. The spring forces T on its ends provide the centripetal acceleration. We have

$$2T\sin\left(\frac{d\theta}{2}\right) = dM\omega^2 R'$$

Using the small-angle approximation $\sin\theta \approx \theta$,

$$T d\theta = dM\omega^2 R'$$
$$T = \frac{dM}{d\theta}\omega^2 R'$$

Since the rubber band has uniform density,

$$\frac{dM}{d\theta} = \frac{M}{2\pi}$$
$$T = \frac{M\omega^2 R'}{2\pi}$$

The change in length of the spring is

$$\Delta L = 2\pi R' - 2\pi R$$

so the tension is

$$T = k\Delta L = 2\pi k(R' - R)$$

Substituting this into the other equation,

$$\begin{aligned} 2\pi k R' - 2\pi k R &= \frac{M\omega^2 R'}{2\pi} \\ 4\pi^2 k R' - M\omega^2 R' &= 4\pi^2 k R \\ R' &= \frac{4\pi^2 k R}{4\pi^2 k - M\omega^2} \end{aligned}$$

so the answer is $\boxed{\text{D}}$.