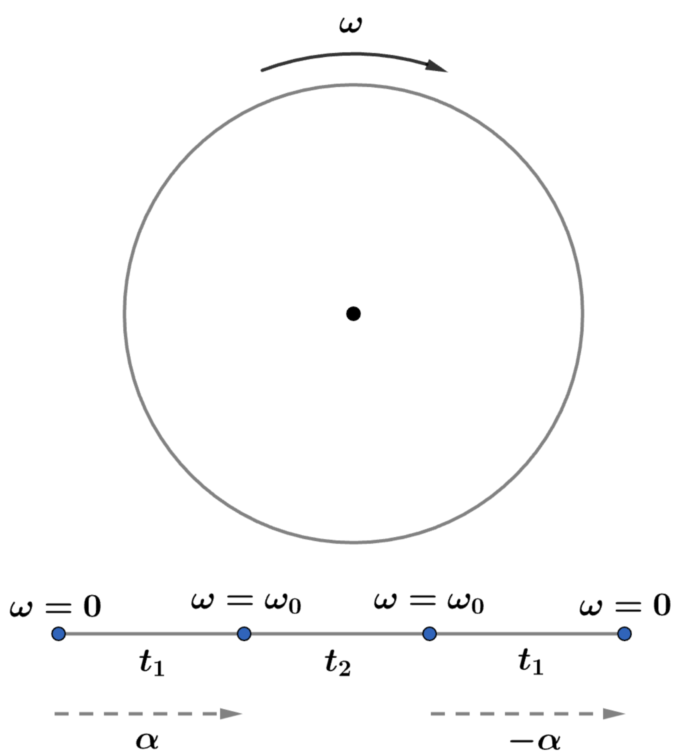
2016 F=ma Exam: Problem 18

Kevin S. Huang



Let t_1 be the time it takes the object to accelerate with α to ω_0 from rest. Let t_2 be the time the object stays rotating at ω_0. Then the time it takes the object to go back to rest is also t_1 since it is decelerating at $-\alpha$ (same magnitude as before). To find t_1, we have

$$\omega_0 = \alpha t_1$$

$$t_1 = \frac{\omega_0}{\alpha} = \frac{\pi/15}{\pi/75}\,\text{s} = 5\,\text{s}$$

To find t_2, we calculate the total angular displacement $\theta_{\text{tot}} = 3(2\pi) = 6\pi$ in terms of t_2. Recall one of our kinematics equations for constant acceleration,

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

where ω_i is the initial angular velocity. During the first t_1 of time, we have

$$\theta_1 = \frac{1}{2}\alpha t_1^2$$

In the next t_2 of time,

$$\theta_2 = \omega_0 t_2$$

In the last t_1 of time,

$$\theta_3 = \omega_0 t_1 - \frac{1}{2}\alpha t_1^2$$

Thus,

$$\theta_{\text{tot}} = \theta_1 + \theta_2 + \theta_3 = \omega_0(t_1 + t_2)$$

$$t_2 = \frac{\theta_{\text{tot}}}{\omega_0} - t_1 = \frac{6\pi}{\pi/15}\,\text{s} - 5\,\text{s} = 85\,\text{s}$$

The total time T taken is

$$T = 2t_1 + t_2 = 2(5\,\text{s}) + 85\,\text{s} = 95\,\text{s}$$

so the answer is $\boxed{\text{E}}$.