2016 F=ma Exam: Problem 14

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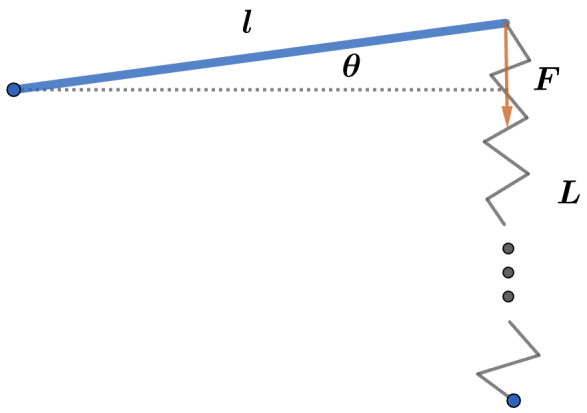
We first find the original oscillation frequency f.



If we displace the rod by a small angle θ, then the spring gets stretched by $x = l\theta$. The spring force F is

$$F = -kx = -kl\theta$$

and the corresponding torque τ is

$$\tau = Fl = -kl^2\theta$$

where we used the small angle approximation $\cos\theta \approx 1$ and the fact that $L \gg l$ to assume the spring force acts downward. Applying Newton's 2nd law $\tau = I\alpha$,

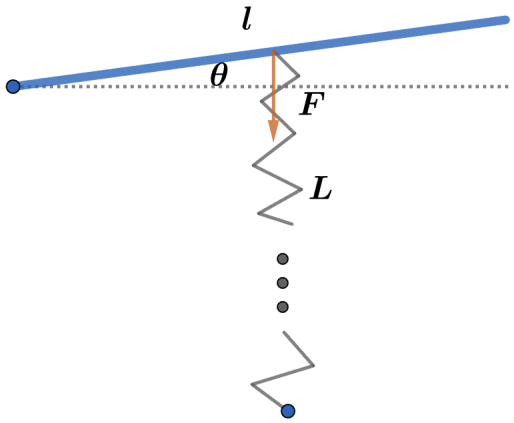
$$-kl^2\theta = I\ddot{\theta}$$
$$\ddot{\theta} = -\frac{kl^2}{I}\theta$$

This is of simple harmonic form ($\ddot{z} = -\omega^2 z$) so we can identify the angular frequency as

$$\omega = \sqrt{\frac{kl^2}{I}}$$

Then the frequency f is

$$f = \frac{\omega}{2\pi} = \frac{l}{2\pi}\sqrt{\frac{k}{I}}$$



If the spring is moved to the midpoint of the rod, then the moment arm l is halved to $l' = l/2$ for the force and torque calculation. The moment of inertia I is unchanged since the rod is still rotating around the same point. Because we found $f \propto l$, we have

$$f' = f/2$$

so the answer is $\boxed{\text{A}}$.