2015 F=ma Exam: Problem 25

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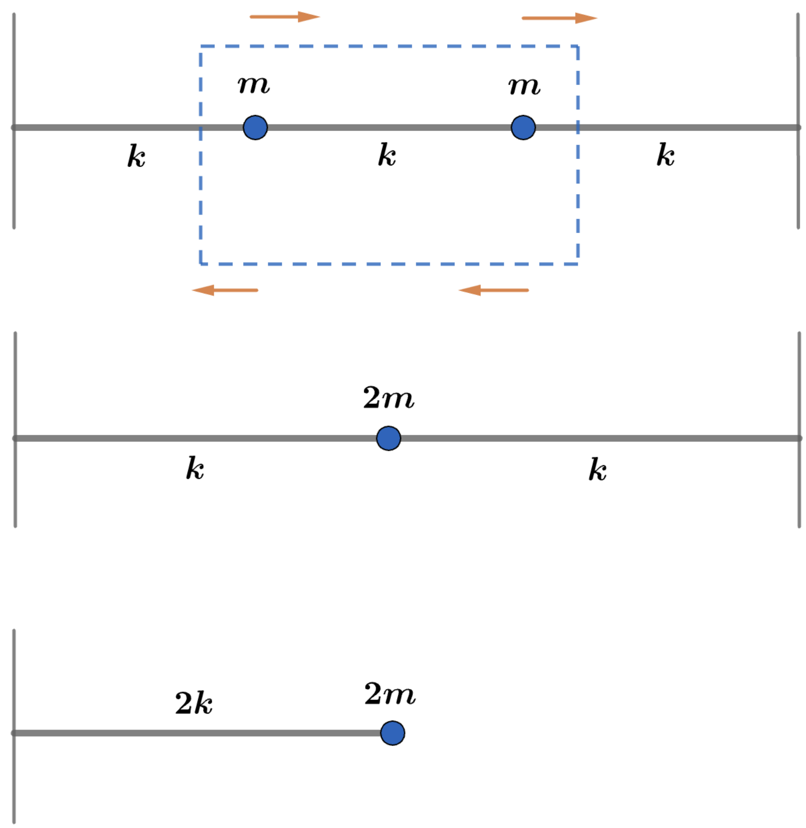
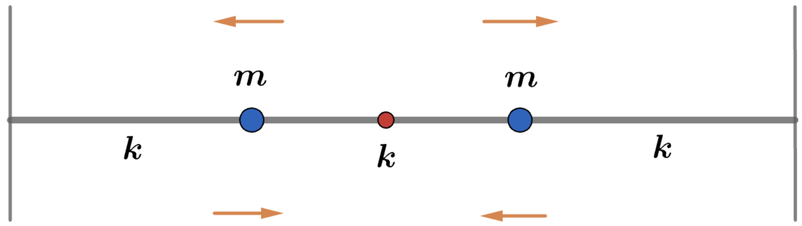
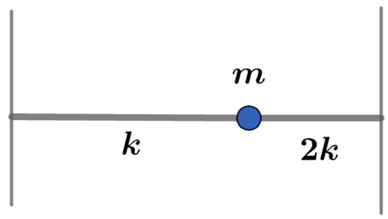
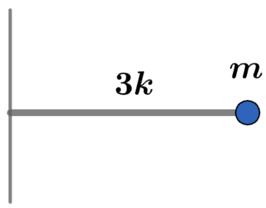


When the two masses oscillate in phase, they always move together so we can equivalently replace them with a single particle of mass $2m$. The two springs on the sides are connected in parallel so we can replace them with a single spring with effective constant $k_e = k + k = 2k$. Now we have a standard mass-spring system which has angular frequency

$$\omega_1 = \sqrt{\frac{2k}{2m}} = \sqrt{\frac{k}{m}}$$







When the two masses oscillate completely out of phase, they always move opposite each other so the midpoint of the center spring is always at rest. Thus, we can attach the midpoint to a wall and just consider one of the masses (the other one is the mirror image). Cutting a spring in half doubles its constant so the middle spring now has $k' = 2k$. The two springs are connected in parallel so we can replace them with a single spring with effective constant $k_e = k + 2k = 3k$. Now we have a standard mass-spring system which has angular frequency

$$\omega_2 = \sqrt{\frac{3k}{m}} = \sqrt{3}\omega_1$$

Thus,

$$\frac{\omega_2}{\omega_1} = \sqrt{3}$$

so the answer is $\boxed{\text{A}}$.