2015 F=ma Exam: Problem 23

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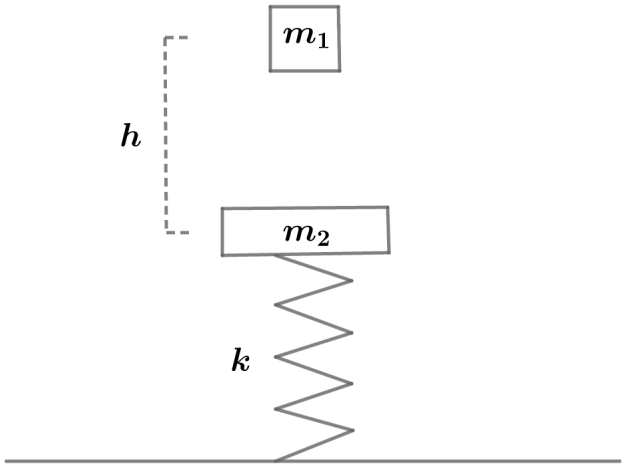


Conserving energy for m_1, we have

$$m_1 gh = \frac{1}{2} m_1 v_1^2$$

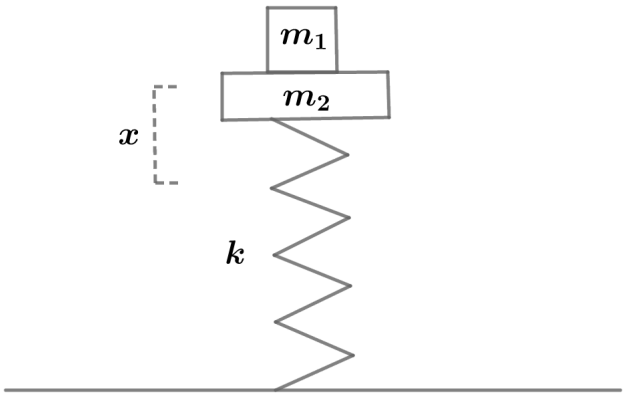
so m_1 hits m_2 with velocity $v_1 = \sqrt{2gh}$. Conserving momentum for the perfectly inelastic collision,

$$m_1 v_1 = (m_1 + m_2) v$$

so both masses move with velocity

$$v = \frac{m_1 v_1}{m_1 + m_2} = \frac{m_1 \sqrt{2gh}}{m_1 + m_2}$$

after the collision.



To find the maximum displacement x of m_2 from its original location, we conserve energy again:

$$E_i = \frac{1}{2}(m_1 + m_2)v^2 = \frac{m_1^2 gh}{m_1 + m_2}$$

$$E_f = \frac{1}{2}kx^2 + m_1 gx$$

so

$$\frac{m_1^2 gh}{m_1 + m_2} = \frac{1}{2}kx^2 + m_1 gx$$

Note that we can treat the vertical spring as a horizontal spring with a shifted equilibrium so the gravitational potential energy of m_2 is already accounted for. Substituting in the given values,

$$25 = 36x^2 + 20x$$

$$36x^2 + 20x - 25 = 0$$

Using the quadratic equation, we find

$$x_1 = 0.6\,\text{m},\ x_2 = -1.1\,\text{m}$$

Hence,

$$\max(|x_1|, |x_2|) = 1.1\,\text{m}$$

so the answer is $\boxed{\text{B}}$.