2015 F=ma Exam: Problem 21

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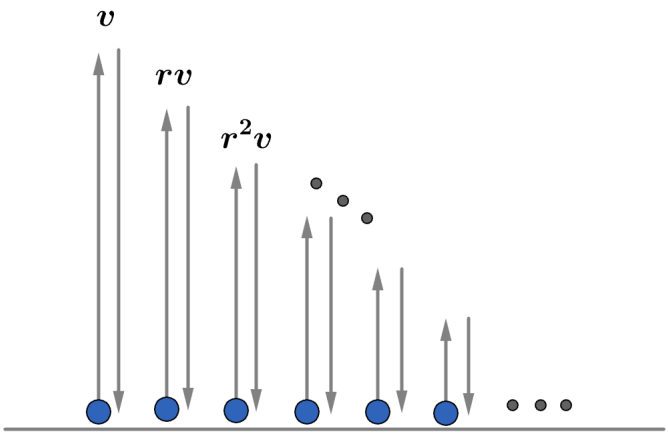

If we launch a ball upward with velocity v, then it takes time

$$v - gt = 0$$
$$t = \frac{v}{g}$$

to reach the top. Coming down takes the same amount of time so the time for the first bounce is

$$T_1 = 2t = \frac{2v}{g}$$

Letting the coefficient of restitution be r, the starting speed for the second bounce is rv. Then the time taken is

$$T_2 = \frac{2rv}{g}$$

For each subsequent bounce, the speed (and hence time) is reduced by a factor of r. The total time is thus

$$T_{\text{tot}} = \sum_{i=1}^{\infty} T_i = \sum_{n=0}^{\infty} \frac{2r^n v}{g} = \frac{2v}{g} \sum_{n=0}^{\infty} r^n = \frac{2v}{g} \frac{1}{1-r}$$

where we used the formula for the sum of an infinite geometric series. Since $v = 50\,\text{m/s}$ and $r = 0.9$,

$$T_{\text{tot}} = \frac{2(50\,\text{m/s}^2)}{(10\,\text{m/s}^2)} \frac{1}{1-0.9} = 100\,\text{s}$$

so the answer is $\boxed{\text{B}}$.