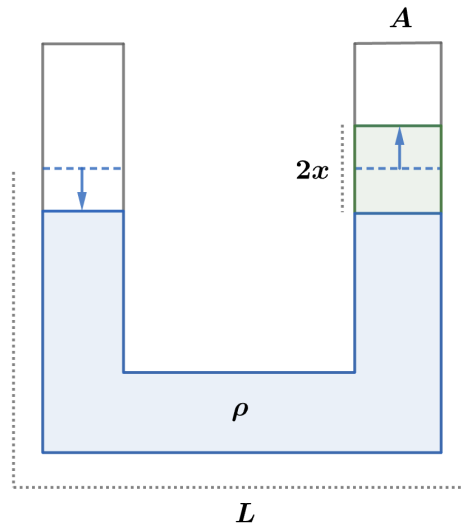# 2015 F=ma Exam: Problem 19

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Suppose we displace the water so that the right side moves up by distance x and the left side moves down by distance x. Then the extra weight on the right side is

$$F = \rho V_{\text{extra}} g = \rho A (2x) g$$

where A is the cross-sectional area of the U-tube. This force accelerates all the water so by Newton's 2nd law,

$$ma = -F = -2\rho A x g$$

where the minus sign accounts for the fact that the force is restoring. Since $m = \rho V = \rho A L$, we have

$$\rho A L a = -2\rho A x g$$
$$a = \ddot{x} = -\frac{2g}{L} x$$

This is of simple harmonic form ($\ddot{z} = -\omega^2 z$) so we can identify the angular frequency as

$$\omega = \sqrt{\frac{2g}{L}}$$

Thus, the frequency is given by

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{2g}{L}}$$

so the answer is $\boxed{A}$.