2015 F=ma Exam: Problem 17

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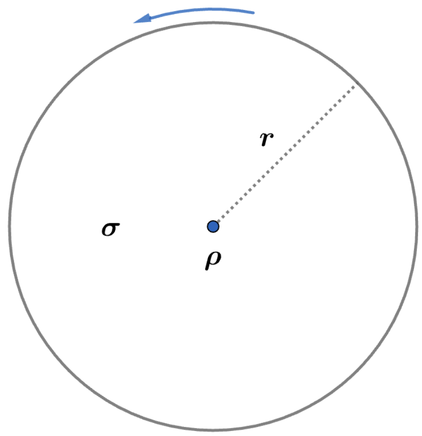

Let λ be the maximum kinetic energy per kilogram that can be stored in the flywheel. If we double the thickness h of the flywheel, that is equivalent to stacking two flywheels on top of each other. Since each has λ energy per kilogram, the combination also has λ energy per kilogram. Hence, λ is independent of h.

We can use dimensional analysis to find $\lambda(r, \rho, \sigma)$. First, the dimensions of each variable are:

$$[\lambda] = \left[\frac{E}{M}\right] = \frac{ML^2/T^2}{M} = \frac{L^2}{T^2}$$
$$[r] = L$$
$$[\rho] = \left[\frac{M}{V}\right] = \frac{M}{L^3}$$
$$[\sigma] = \left[\frac{F}{A}\right] = \frac{ML/T^2}{L^2} = \frac{M}{LT^2}$$

We need 1 power of σ to have the correct number of powers of T:

$$[\sigma^1] = \frac{M}{LT^2}$$

We need -1 powers of ρ to cancel out the mass dependence:

$$\left[\frac{\sigma}{\rho}\right] = \frac{M}{LT^2}\frac{L^3}{M} = \frac{L^2}{T^2}$$

This has the same dimensions as λ so there is no r dependence. Thus,

$$\lambda = \frac{\alpha\sigma}{\rho}$$

where α is a dimensionless constant, so the answer is $\boxed{\text{E}}$.