2014 F=ma Exam: Problem 22

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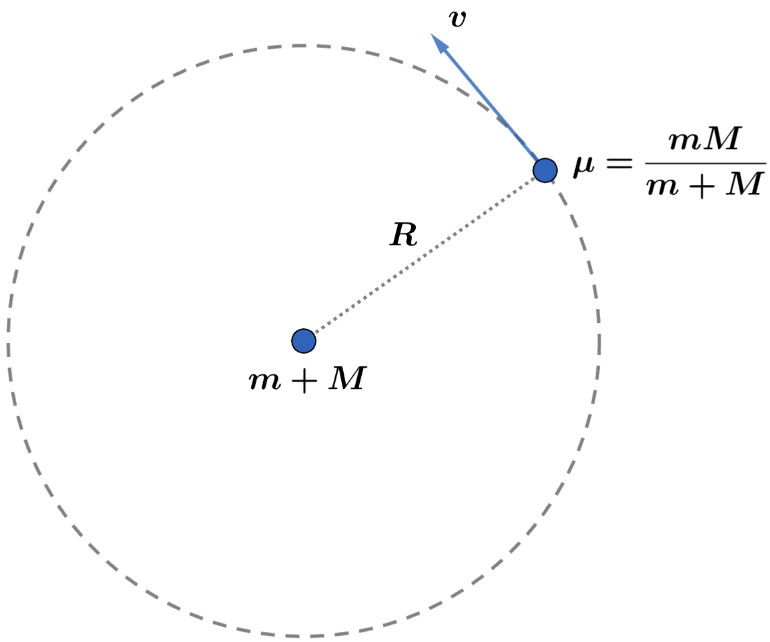

To simplify calculating the angular momentum and period, we use the concept of reduced mass to turn the two-body problem of masses m and M interacting via gravity into the equivalent one-body problem of a particle orbiting a fixed source. Recall the reduced mass of m and M is given by

$$\mu = \frac{mM}{m+M}$$

In order for the single particle of mass μ to experience the same force $F = GMm/R^2$ as the gravity between m and M, the fixed source must have mass $m + M$.

Now, we go through the possible choices:

- A) Not correct since the gravitational force is

$$F = \frac{GMm}{R^2}$$

If $m \to m - \delta m$ and $M \to M + \delta m$ then

$$mM \to (m - \delta m)(M + \delta m) = mM - (M - m)\delta m - (\delta m)^2 < mM$$

so the gravitational force decreases.

- B) Not correct since the gravitational force decreases.
- C) Not correct since the angular momentum is

$$L = \mu v R = \frac{mM}{m+M}\sqrt{\frac{G(m+M)}{R}}R = mM\sqrt{\frac{GR}{m+M}}$$

  where we used the velocity of a circular orbit $v_{\text{circ}} = \sqrt{GM_{\text{source}}/R}$. Since mM decreases while $m+M$ stays constant, the angular momentum decreases.
- D) Not correct since the angular momentum decreases.
- E) Correct since the period is

$$T^2 = \frac{4\pi^2}{G(m+M)}R^3$$

  where we used Kepler's 3rd law with $M_{\text{source}} = m + M$. Since $m+M$ stays constant, the period remains constant.

Thus, the answer is $\boxed{\text{E}}$.