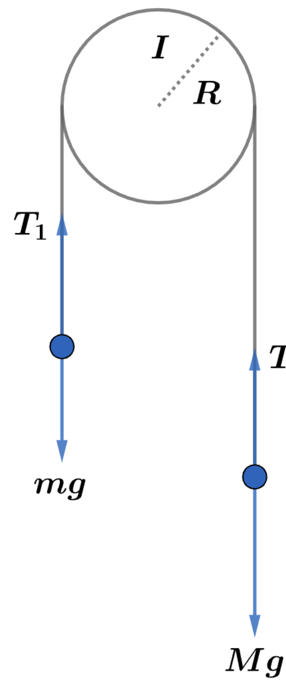
2014 F=ma Exam: Problem 21

Kevin S. Huang



We first find the acceleration in an Atwood machine where the pulley has moment of inertia I. Applying Newton's 2nd law to each mass and the pulley,

$$T_1 - mg = ma$$

$$Mg - T_2 = Ma$$

$$\tau = T_2 R - T_1 R = I\alpha$$

Assuming there is no slipping $\alpha = a/R$, we have

$$T_2 - T_1 = \frac{Ia}{R^2}$$

Adding this equation and the first two equations yields

$$Mg - mg = ma + Ma + \frac{Ia}{R^2}$$

so the acceleration is

$$a = \frac{M - m}{m + M + I/R^2} g$$

The ratio between the accelerations in systems A and B is

$$\frac{a_A}{a_B} = \frac{(M-m)g}{m+M+I_A/R^2}\frac{m+M+I_B/R^2}{(M-m)g} = \frac{(m+M)R^2+I_B}{(m+M)R^2+I_A}$$

Since pulley A has mass $m+M$ and is a uniform disk,

$$I_A = \frac{1}{2}(m+M)R^2$$

The mass of a disk is proportional to its area $m \propto r^2$ so a disk with radius $r/2$ has mass $m/4$. Thus, the moment of inertia of the removed hole is

$$I_{\text{hole}} = \frac{1}{2}\frac{(m+M)}{4}\left(\frac{R}{2}\right)^2 = \frac{I_A}{16}$$

so for pulley B,

$$I_B = I_A - I_{\text{hole}} = \frac{15}{16}I_A = \frac{15}{32}(m+M)R^2$$

Substituting this into the ratio of accelerations,

$$\frac{a_A}{a_B} = \frac{(m+M)R^2+(15/32)(m+M)R^2}{(m+M)R^2+(1/2)(m+M)R^2} = \frac{1+(15/32)}{1+(1/2)} = \frac{47/32}{3/2} = \frac{47}{48}$$

so the answer is $\boxed{\text{A}}$.