2014 F=ma Exam: Problem 14

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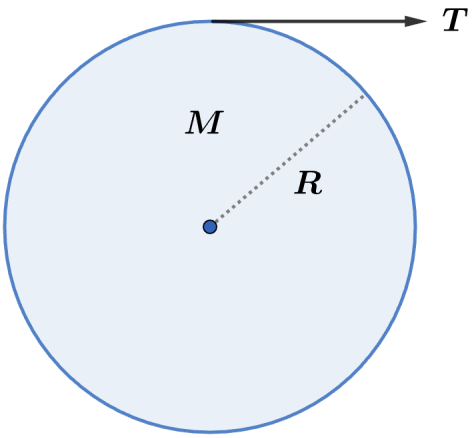

By Newton's 2nd law, we have

$$T = Ma$$
$$\tau = TR = I\alpha$$

so $a = T/M$ and $\alpha = TR/I$. The kinetic energies are given by

$$K_{\text{trans}} = \frac{1}{2}Mv^2 = \frac{1}{2}M(at)^2$$
$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}I(\alpha t)^2$$

Then by canceling out common factors,

$$\frac{K_{\text{trans}}}{K_{\text{tot}}} = \frac{K_{\text{trans}}}{K_{\text{trans}} + K_{\text{rot}}} = \frac{Ma^2}{Ma^2 + I\alpha^2} = \frac{M(1/M)^2}{M(1/M)^2 + I(R/I)^2} = \frac{\frac{1}{M}}{\frac{1}{M} + \frac{R^2}{I}}$$

Multiplying the numerator and denominator by MI,

$$\frac{K_{\text{trans}}}{K_{\text{tot}}} = \frac{I}{I + MR^2}$$

so the answer is $\boxed{\text{D}}$.