2014 F=ma Exam: Problem 13

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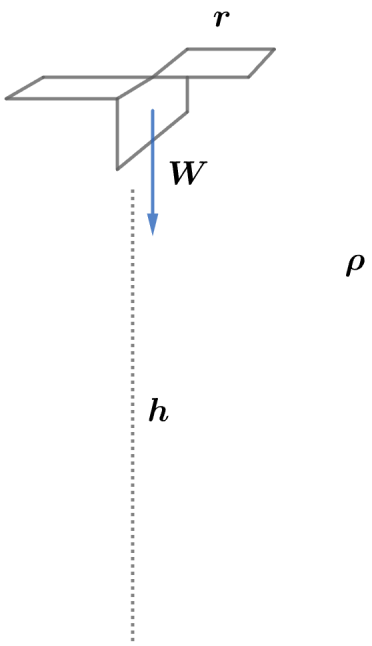

From the previous problem, we found $T = khr^\beta \rho^\delta W^\omega$. Taking the dimensions of both sides,

$$[T] = [h][r]^\beta [\rho]^\delta [W]^\omega$$

Using the fact that $[\rho] = M/L^3$ and $[W] = ML/T^2$,

$$T = L^1 L^\beta \left(\frac{M}{L^3}\right)^\delta \left(\frac{ML}{T^2}\right)^\omega$$

Counting powers of T:

$$1 = -2\omega$$
$$\omega = -1/2$$

Counting powers of M:

$$0 = \delta + \omega$$
$$\delta = -\omega = 1/2$$

Counting powers of L:

$$0 = 1 + \beta - 3\delta + \omega$$
$$\beta = 3\delta - \omega - 1 = -4\omega - 1 = 2 - 1 = 1$$

Since $\beta = 1$, the answer is $\boxed{\text{D}}$.