2013 F=ma Exam: Problem 17

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We go to the rotating frame where the rod is at rest and include the (fictitious) centrifugal force directed outwards on all objects. Then, the external forces on our system are gravity from the planet and the centrifugal force. The gravitational force is given by

$$F_g = \frac{GMm}{r^2}$$

where M is the mass of the planet and r is the distance to the planet's center. We see that gravity is stronger on the inner mass since it is closer to the planet:

$$F_{g1} > F_{g2}$$

The centrifugal force is given by

$$F_c = m\omega^2 r$$

where ω is the angular velocity of the rod in the original reference frame. We see that the centrifugal force is stronger on the outer mass since it is further away:

$$F_{c2} > F_{c1}$$

Balancing forces,

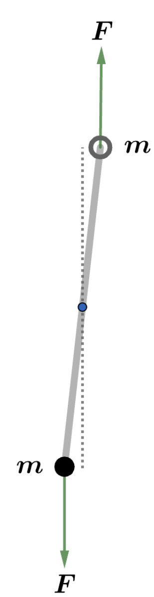
$$F_{c1} + F_{c2} = F_{g1} + F_{g2}$$

Since $F_{c1} < F_{c2}$, F_{c1} is less than their average,

$$F_{c1} < \frac{F_{c1} + F_{c2}}{2} = \frac{F_{g1} + F_{g2}}{2} < F_{g1}$$

where we used the fact that $F_{g2} < F_{g1}$ for the last inequality. Since $F_{g1} > F_{c1}$, the net external force on the inner mass $F = F_{g1} - F_{c1} > 0$ is directed inwards. Similarly, the net external force (also F) on the outer mass is directed outwards.

Thus, the rod must be in tension. To check stability, we displace the rod from equilibrium by rotating it through a small angle. Since the external forces on the masses point away from the center of the rod, the torque is restoring so the equilibrium is stable.



Thus, the answer is $\boxed{\text{B}}$.