2013 F=ma Exam: Problem 15

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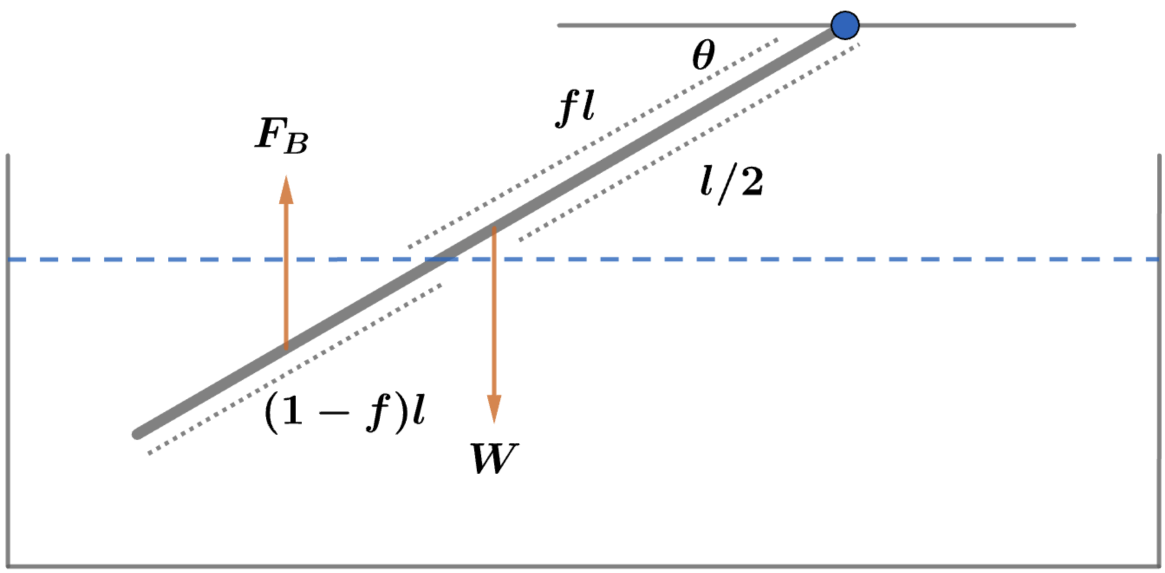

We choose the hinge as our pivot point and balance torques on the rod. Only the weight of the rod and the buoyant force contribute to the torque. The weight W acts downward at the CM of the rod, distance $l/2$ away from the pivot. The buoyant force F_B acts upward at the CM of the underwater portion. If f is the fraction of the rod above water, then F_B acts at a distance

$$d = fl + \frac{(1-f)l}{2} = \frac{(1+f)l}{2}$$

away from the pivot. Balancing torques,

$$W\left(\frac{l}{2}\cos\theta\right) = F_B\left[\frac{(1+f)l}{2}\cos\theta\right]$$
$$W = F_B(1+f)$$

The weight is given by

$$W = m_{\text{rod}}g = \rho_{\text{rod}}Alg$$

where A is the cross-sectional area of the rod. By Archimedes' principle,

$$F_B = \rho_{\text{water}}V_{\text{sub}}g = \rho_{\text{water}}A(1-f)lg$$

Substituting into the earlier equation,

$$\rho_{\text{rod}} A l g = \rho_{\text{water}} A (1 - f) l g (1 + f)$$
$$\rho_{\text{rod}} = \rho_{\text{water}} (1 - f^2)$$

Since $\rho_{\text{rod}} = \frac{5}{9}\rho_{\text{water}}$,

$$\frac{5}{9} = 1 - f^2$$

Solving for f,

$$f^2 = \frac{4}{9}$$
$$f = \frac{2}{3}$$

so the answer is $\boxed{\text{D}}$.