2013 F=ma Exam: Problem 12
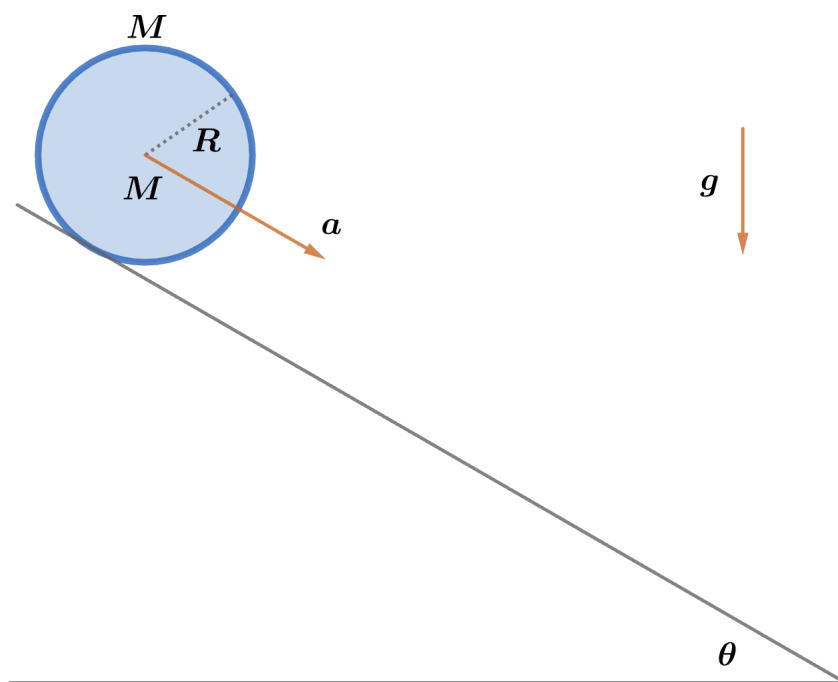
Kevin S. Huang

Recall the acceleration a of an object rolling without slipping down an incline of angle θ is

$$a = \frac{g \sin \theta}{1 + \beta}$$

where the object has moment of inertia $I = \beta m r^2$. We will find the effective β for our composite object of a spherical shell containing a frictionless fluid. Since the shell and fluid both have mass M, this object has mass $2M$. A frictionless fluid does not have rotational inertia so only the shell contributes to the moment of inertia $I = \frac{2}{3} M R^2$. Thus,

$$\beta = \frac{I}{m r^2} = \frac{\frac{2}{3} M R^2}{(2M) R^2} = \frac{1}{3}$$

$$a = \frac{g \sin \theta}{1 + (1/3)} = \frac{3}{4} g \sin \theta$$

so the answer is $\boxed{\text{B}}$.