# 2012 F=ma Exam: Problem 25

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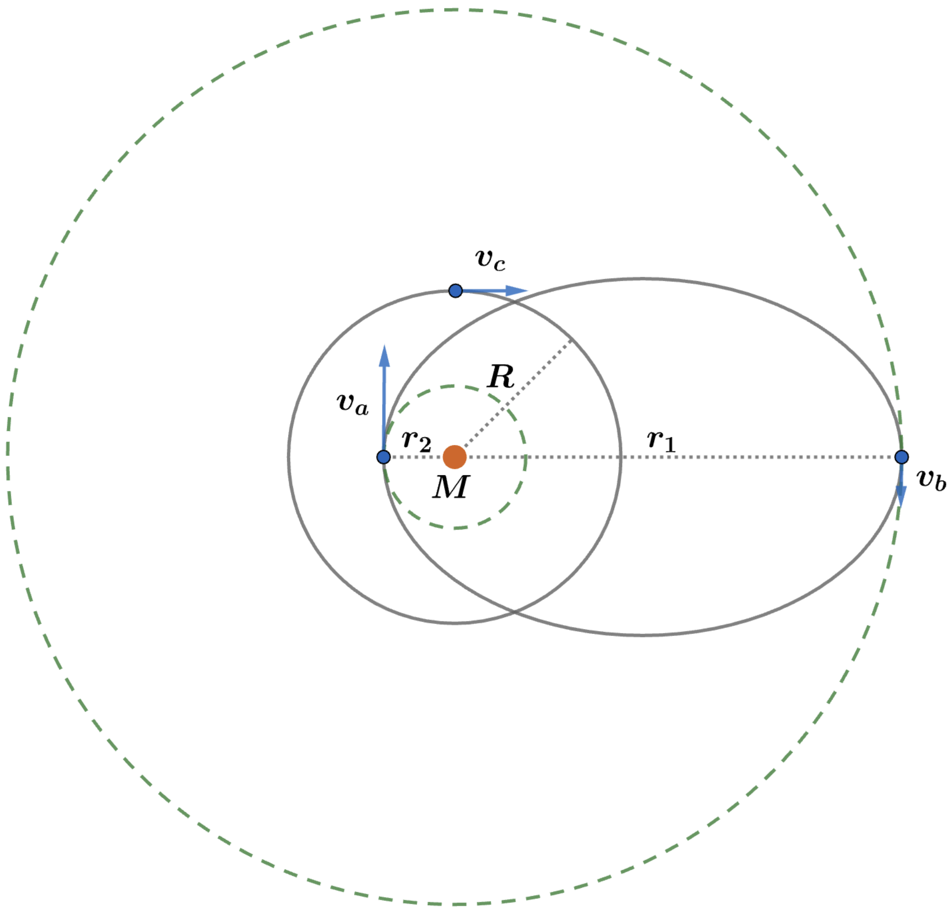


Conserving angular momentum between the apogee and perigee of the ellipse, we have

$$mv_b r_1 = mv_a r_2$$
$$\frac{v_a}{v_b} = \frac{r_1}{r_2}$$

Since $2R \leq r_1 \leq 3R$ and $R/3 \leq r_2 \leq R/2$, we can bound

$$4 = \frac{2R}{R/2} \leq \frac{v_a}{v_b} \leq \frac{3R}{R/3} = 9$$
$$4v_b \leq v_a \leq 9v_b \tag{1}$$

Recall the velocity of a circular orbit of radius $r$ is given by

$$v_{\text{circ}} = \sqrt{\frac{GM}{r}}$$

We can relate $v_b$ to $v_c = \sqrt{GM/R}$ by considering a circular orbit of radius $r_1$. Then

$$v_b < \sqrt{\frac{GM}{r_1}} \leq \sqrt{\frac{GM}{2R}} = \frac{v_c}{\sqrt{2}}$$

since the ellipse is contained in the circle and $r_1 \geq 2R$. We can also relate $v_a$ to $v_c$ by considering a circular orbit of radius $r_2$. Then

$$v_a > \sqrt{\frac{GM}{r_2}} \geq \sqrt{\frac{GM}{R/2}} = \sqrt{2}v_c$$

since the circle is contained in the ellipse and $r_2 \leq R/2$. Combining these two equations,

$$v_b < \frac{v_c}{\sqrt{2}} < v_c < \sqrt{2}v_c < v_a \tag{2}$$

We now go through the possible choices:

- A) $v_b > v_c > 2v_a$
  Not correct because $v_b < v_c$ from equation 2.

- B) $2v_c > v_b > v_a$
  Not correct because $v_b < v_a$ from equation 2.

- C) $10v_b > v_a > v_c$
  Correct because $v_a \leq 9v_b < 10v_b$ from equation 1 and $v_c < v_a$ from equation 2.

- D) $v_c > v_a > 4v_b$
  Not correct because $v_c < v_a$ from equation 2.

- E) $2v_a > \sqrt{2}v_b > v_c$
  Not correct because $\sqrt{2}v_b < v_c$ from equation 2.

Thus, the answer is $\boxed{\text{C}}$.