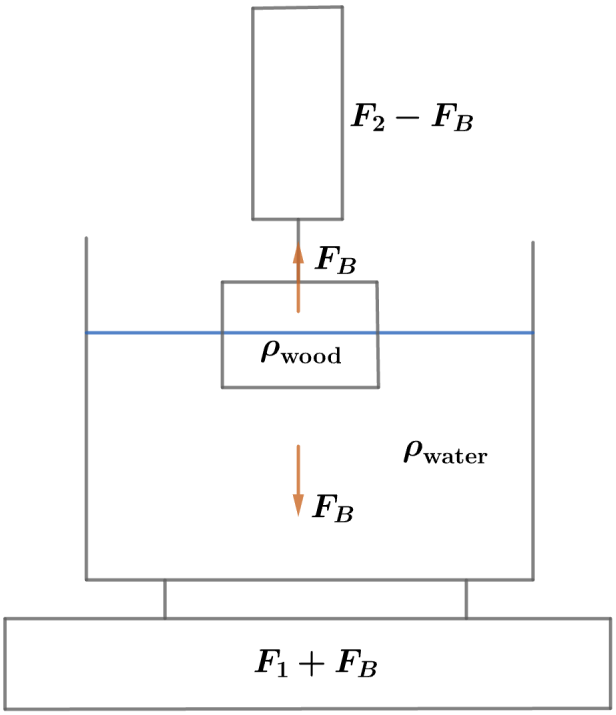
2012 F=ma Exam: Problem 20

Kevin S. Huang

Note that the scales determine mass by dividing their measured force by g. Thus, in the beginning, the first scale measures the normal force on itself as $F_1 = M_1 g$. The second scale measures its tension as $F_2 = M_2 g$.



Once the block is submerged in the water, the buoyant force F_B acts upward on the block reducing the tension to $T = F_2 - F_B$. By Newton's 3rd law, the block also exerts a force F_B down on the water so the normal force on the bottom increases to $N = F_1 + F_B$.

By Archimedes' principle, we have

$$F_B = \rho_{\text{water}} \left(\frac{V}{2} \right) g$$

where V is the volume of the block. Since the block has mass M_2,

$$M_2 = \rho_{\text{wood}} V$$
$$V = \frac{M_2}{\rho_{\text{wood}}}$$

Substituting this back into F_B,

$$F_B = \frac{\rho_{\text{water}}}{\rho_{\text{wood}}} \frac{M_2 g}{2}$$

Then the mass measured by the first scale is

$$M_1' = \frac{N}{g} = \frac{F_1 + F_B}{g} = M_1 + \frac{\rho_{\text{water}}}{\rho_{\text{wood}}} \frac{M_2}{2} = 45\,\text{kg} + 10\,\text{kg} = 55\,\text{kg}$$

The mass measured by the second scale is

$$M_2' = \frac{T}{g} = \frac{F_2 - F_B}{g} = M_2 - \frac{\rho_{\text{water}}}{\rho_{\text{wood}}} \frac{M_2}{2} = 12\,\text{kg} - 10\text{kg} = 2\,\text{kg}$$

so the answer is $\boxed{\text{E}}$.