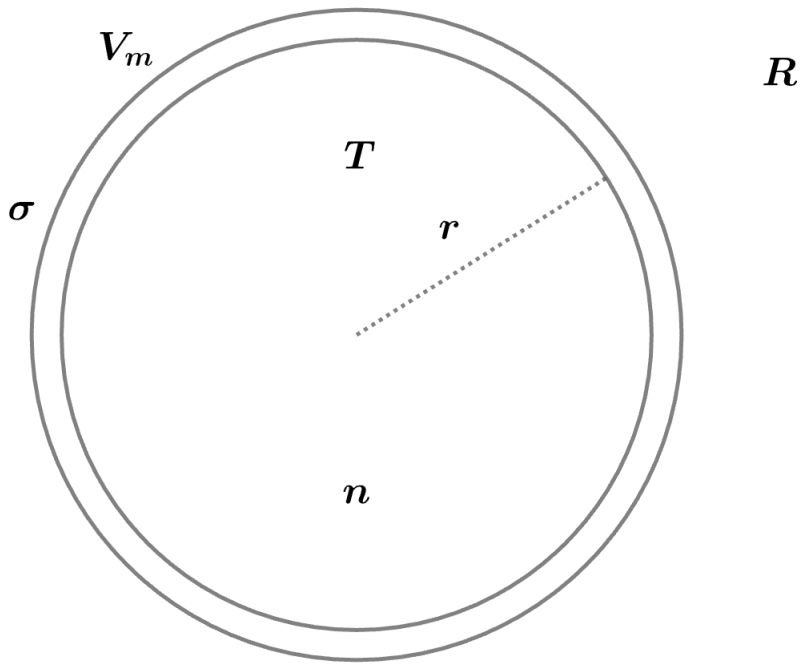
2011 F=ma Exam: Problem 21

Kevin S. Huang



We can find the form of $n(V_m, T, R, \sigma)$ via dimensional analysis. We have the units of all variables:

$$[n] = \text{mol}$$
$$[V_m] = \text{m}^3$$
$$[T] = \text{K}$$
$$[R] = \frac{\text{J}}{\text{K} \cdot \text{mol}}$$
$$[\sigma] = \frac{\text{N}}{\text{m}^2}$$

Since $[n] = \text{mol}$, we must have $n \propto R^{-1}$. Then

$$[R^{-1}] = \frac{\text{mol} \cdot \text{K}}{\text{J}}$$

so we have to divide by T to cancel out K and obtain $n \propto R^{-1}T^{-1}$. Then

$$[R^{-1}T^{-1}] = \frac{\text{mol}}{\text{J}}$$

so we have to get rid of units of J. If we multiply σ and V_m, we find

$$[\sigma V_m] = \mathrm{N} \cdot \mathrm{m} = \mathrm{J}$$

so we can just multiply σV_m to the earlier expression. Then

$$[\sigma V_m R^{-1} T^{-1}] = \mathrm{mol}$$

as required. Therefore,

$$n \propto \frac{\sigma V_m}{RT}$$

so the answer is $\boxed{\text{A}}$.