2007 F=ma Exam: Problem 32

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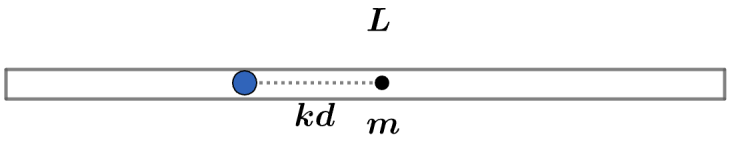

Recall the angular frequency of a physical pendulum is given by

$$\omega = \sqrt{\frac{mgs}{I}}$$

where s is the distance between the CM and the pivot point and I is the moment of inertia around the pivot point. In our case, we have

$$s = kd$$
$$I = md^2 + m(kd)^2 = (1 + k^2)md^2$$

using $I_{\text{rod}} = md^2$ (by definition of d in the problem) and the parallel-axis theorem. Then

$$\omega = \sqrt{\frac{mgkd}{(1 + k^2)md^2}} = \sqrt{\frac{k}{1 + k^2}}\sqrt{\frac{g}{d}}$$

Since we are given

$$\omega = \beta\sqrt{\frac{g}{d}}$$

we can identify

$$\beta = \sqrt{\frac{k}{1 + k^2}}$$

so the answer is $\boxed{\text{E}}$.