STATEMENT OF PROBLEM: The fundamental scientific problem this project will address is that of the joint measurement of trapped electronic spins via ballistic electrons in a nearby conductance channel. A joint measurement is a measurement of a two-spin or multi-spin operator that does not reveal the individual state of each spin. Joint measurements are of fundamental significance in quantum information processing. A procedure that acts as a joint measurement can be used to facilitate entanglement or the fault-tolerant encoding of information into multiple physical qubits.

OVERVIEW: Recent advances in the fabrication and tenability of high-quality semiconductor heterostructures have allowed the experimental observation of various types of electronic spin qubits [1-5]. These systems are envisioned as being useful for the construction of a quantum computer, but are also interesting on a fundamental level. Even systems of only a few spins will provide a fascinating window into the emergence of many-body entanglement. However, the means of entangling the spins is currently limited to either superexchange via direct tunnel coupling, which is extremely short ranged and requires dense, tunable, technically challenging gate arrays, or Coulomb interaction, which is weak and susceptible to noise at long distance. Likewise, measurements are limited to single-spin probes, proceeding via tunnel coupling to a reservoir or capacitive coupling to a quantum point contact. <u>This project seeks to open an entirely new direction in the field</u> by theoretically exploring the idea of performing a joint measurement on an array of spatially separated spins by measuring the current in one or more proximal conductance channels coupled to the spins. (The coupling employs a momentary "spin-to-charge" conversion, essentially just using Pauli blocking, similar to how "singlet-triplet" qubits are already coupled to a quantum point contact for readout [1].) Such joint measurements can be used to entangle non-interacting spins and furthermore form the building blocks of fault-tolerant error correction proposals, but are not currently experimentally accessible in solid state spin qubit systems.

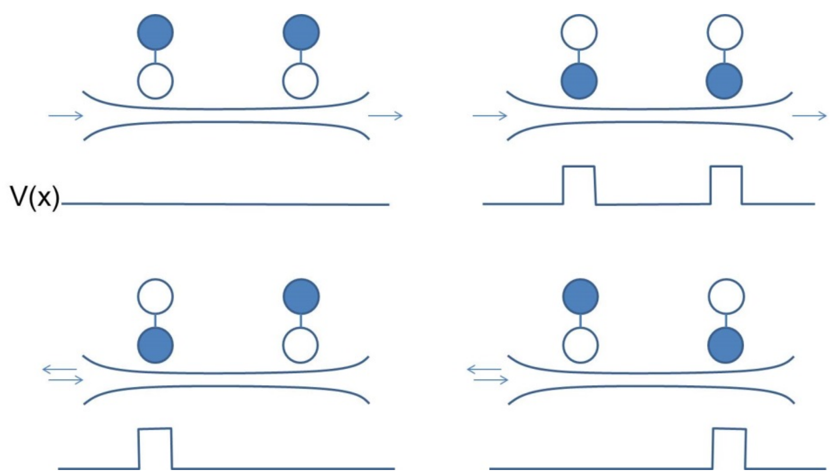

Figure 1: Double-quantum-dot charge qubits capacitively coupled to a conductance channel create a scattering potential that depends on the joint state of the qubits. Open (shaded) circles here represent empty (charged) dots, but Pauli blocking can be used to translate the idea to spin qubits.

The central idea here is sketched in Figure 1 and the recipe is very simple:

Two unequal reservoirs. These are taken to be two disconnected regions of a two-dimensional electron gas (2DEG) residing near the interface of a semiconductor heterostructure. They have different Fermi energies, controlled via voltage leads, so as to permit net charge transport.

One conductance channel. This is taken to be a quasi-one-dimensional constriction in the background 2DEG, connecting the two reservoirs. Alternatively, a tunnel-coupled nanowire may be used.

Two spin qubits. (Add more as desired.) A quantum dot contains a single electron, whose spin projection forms the qubit. To allow selective electrostatic coupling to the spin qubit, add gated tunnel coupling to another dot whose energy levels are Zeeman split so as to allow spin-selective tunneling.

Alternatively, and even better for other reasons, one may use a singlet-triplet qubit formed by two electrons in a double quantum dot with tunable bias. In any event, the result is that upon adjusting a tunnel barrier or bias voltage, the charge density shifts in a spin-dependent way so that information is temporarily stored in the charge configuration rather than isolated in the spin degree of freedom. In fact, if decoherence times were not a concern, one could simply substitute charge qubits. The proposed research will be undertaken with singlet-triplet qubits in mind, but it could easily be adapted for other types of qubits.

Combining these ingredients together, the electrostatic couplings between the qubits and the conductance channel create a multi-barrier scattering potential for the transport electrons that depends on the spin states of the trapped electrons. Thus, there is a set of transmission resonances that depend on the joint state of all of the trapped spins. For example, in the simplest case of two spins and transport electrons incident at a given energy onto the resulting double-barrier potential, there is a resonant symmetric barrier height such that the transmission is 100%. By tuning the couplings (via the voltage gates) such that a barrier is zero if its corresponding spin is down, and at the resonant height if its corresponding spin is up, a measurement of conductance will give only 0 or 1, although there are four possible total spin states. The measurement only indicates whether or not the spins are aligned, without distinguishing between whether they are both up or both down. It does not collapse the wavefunction within a given parity subspace.

BACKGROUND: We have seen that, in principle, one can use a resonant tunneling scheme to perform a joint measurement on the spins via conductance. This ability, along with an ancilla spin, can be used to entangle spatially separated spins, an idea that is familiar in the linear optics community from the Knill-Laflamme-Milburn proposal [6]. This is remarkable since the spins need not interact with each other at all – joint measurement provides the nonlinearity. Previous attempts to exploit this property in quantum dots required coupling charges to a single quantum point contact [7,8] or ancilla [9], requiring the dots to be very close to each other, in which case one could also presumably use direct coupling. In the current proposal, by exploiting the nonzero phase coherence length and quantum mechanical tunneling resonances of the transport electrons, one can consider small systems of spins separated by several microns.

Furthermore, and perhaps even more interestingly, the backbone of quantum error correction is the parity measurement, with the four-qubit joint measurement being of particular interest for Kitaev's toric code [10], Steane's 7-qubit code [11], and the perfect five-qubit code [12, 13]. These joint measurements are typically imagined as being performed by a series of two-qubit operations between data qubits and ancillas, but the direct approach proposed here would eliminate that laborious process and perform the joint measurement in a single-shot, in addition to relaxing the usual physical condition that ancillas and data qubits be in close proximity.

Both resonant tunneling and measurement-based quantum information processing are well-established ideas that have been studied extensively. However, to my knowledge neither has ever been placed in the context of the other. This combination of simple ideas immediately raises interesting and fundamental theoretical questions and lends itself to several extensions. For example, what is the practical limit on detection efficiency? Is it possible to generalize the arrangement so that the conductance measurement performs a projection onto the even/odd parity subspaces of an N-spin system? (Under the simplest set of assumptions, the answer is yes, as we will show in the Preliminary Results section.) What kind of decreases in algorithmic complexity could this ability facilitate? The results will have immediate applications in quantum information as well as fundamental studies of many-body entanglement, and this idea could prove to be a great advance. However, the devil is in the details – which is fortunate, because it gives us something to calculate! There are many details requiring careful consideration that are actually very interesting in their own right. For instance, the result of various imperfections will be to introduce some amount of *welcher-weg* information, allowing one to begin to distinguish between individual states within the specified parity subspace and causing the joint measurement (or "syndrome," in the language of quantum error correction) to gradually transition into a mere measurement of the individual spins. Such details form the substance of the research plan of this proposal, discussed below. <u>The detailed study proposed here must first be performed before this type of joint measurements can actually be implemented in experiment.</u>

RESEARCH OBJECTIVES:

1) Calculate the requirements in principle for current measurements to distinguish between two or more subspaces of the multi-spin system without resolving the individual states. This involves modeling the potential, accounting for the width of the resonance, the energy spread of the injected electrons, the number of incident electrons per pulse, etc.

2) Calculate the sensitivity of the joint measurement to realistic imperfections. This involves considering the effects of imprecise gate voltages, background 1/f charge noise, decoherence during measurement, etc.

3) Consider the capabilities of multi-qubit joint measurements more generally. Outside the specific physical context, while it is well known how to maximally entangle pairwise by using two-qubit joint measurements and ancillas, it will be interesting to consider how to efficiently create highly entangled multi-qubit states when one has direct access to multi-qubit joint measurements.

PLAN OF PROCEDURE: The approach is to begin with <u>transfer matrix</u> calculations for a 1D model scattering potential [14]. This is extremely straightforward, and some preliminary results are given below for the simplest possible model of symmetric square barriers. This heuristic approach will give qualitative insight, but the plane wave eigenstates of the constant potentials should actually be replaced by Airy functions for a linear potential in the presence of a symmetry-breaking voltage bias across the channel [15] and the current obtained using the Tsu-Esaki formula [16]. The next step is to use a realistic model formed by <u>numerical solution of Poisson's equation</u> in a gated semiconductor heterostructure geometry. The one-dimensional approximation must also be justified, and if more than one channel is open in the physically realistic system, then that will also have to be accounted for. A final level of refinement will be to use a coupled Poisson-Schrodinger solver to obtain the potential as a function of gate voltages and spin state. This will include the effects of gate geometry through rudimentary COMSOL device modelling.

The important questions to address in this context is what the conditions on the inter-qubit distances, barrier heights, and incident energies are in order to allow good distinguishability in conductance between even and odd parity states without distinguishing between individual states in the subspace. Also, the effects of disorder will be analyzed. Static disorder in the barriers may be calibrated away, but for higher-frequency components of noise it will be useful to find regions of parameter space that offer natural protection against fluctuations. It may also become important to take into account the transient behavior of the system via a numerical solution for the time-dependence of the transport [17].

Another important question is that of decoherence during the measurement. First of all, one must consider the decoherence of the qubits since the temporary partial transfer of information from spin into charge in order to couple to the transport channel also makes the qubit temporarily vulnerable to dephasing charge noise. There are a couple of ways to address this. One is to consider the case of extremely short distance between the qubit and the transport channel so that the coupling to the channel is much larger than the typical coupling to noise. In that case one can simply tilt the double-dot potential only slightly so that very little charge density is shuttled, with correspondingly small spin-dependent charge dipole to allow decoherence while still producing a transport channel potential barrier of sufficient height. Note that this is the basic approach already demonstrated for direct capacitive coupling of two qubits [18]. Another way to address decoherence would be to consider a pulse sequence for the double-dot tilt such that the entire operation carries out a dynamical decoupled evolution. I have expertise with this sort of approach in recent years (see, e.g., Refs [19-21]), and early experimental implementation has been successful [22]. Secondly, one must consider the onset of dephasing of the transport electrons when one begins to consider larger inter-qubit distances or multi-qubit parity checks. These considerations will show the boundaries on the application of this scheme.

Anticipated timeline:

Year 1: Complete analysis for heuristic square-barrier potential model, as well as simple linear models, including estimates of the sensitivity of the parity measurement to static disorder and the requirements for avoiding a fatal amount of decoherence during the operation. (Preliminary results quoted below.) Begin fully numerical transfer matrix calculations for potential from solution of Poisson's equation.

Year 2: Complete calculations for realistic potential. Add detailed device modeling. Consider high-frequency components of disorder as well as transient behavior of system.

Year 3: Examine transition between faithful and unfaithful syndromes. Consider capabilities of direct N-qubit parity measurements. (Also, other interesting unforeseen theoretical questions that inevitably emerge over the prior two years!)

PRELIMINARY RESULTS: Here we show some back-of-the-envelope style calculations for a simplistic model that shows the feasibility of the proposal. Consider a 2DEG in a GaAs/AlGaAs heterostructure with a disorder-free constriction containing only one conductance channel, assuming that the electrons entering the channel from the reservoir have a definite energy that is tunable on the micro-volt scale. A typical length scale in systems of lateral gate-defined quantum dots is $\sim 100\ nm$. Assume that the spins are initially positioned in dots roughly $\ell = 200\ nm$ away from the channel and that turning up the inter-dot bias results in a fraction $\frac{\delta e}{e}$ of the charge density tunneling into a dot $\ell' = 100\ nm$ away from the channel, conditioned on the spin state. In a simple-minded picture that ignores direct coupling between the bias voltage gate and the conductance channel, the barrier potential induced by such a shift is $V(x) = \frac{\delta e}{e}\frac{e^2}{4\pi\epsilon}\left(\frac{1}{\sqrt{x^2+\ell'^2}} - \frac{1}{\sqrt{x^2+\ell^2}}\right)$. Further approximate this as rectangular (not because it is necessarily good, but because it is easy!), with height $V_0 \cong \frac{e^2}{4\pi\epsilon}\left(\frac{1}{\ell'} - \frac{1}{\ell}\right) \cong 550\ \mu eV$ for 100% charge tunneling and width of $a = 200\ nm$ to match the full width at half maximum of the smooth potential. Assume also that the system is perfectly symmetric.

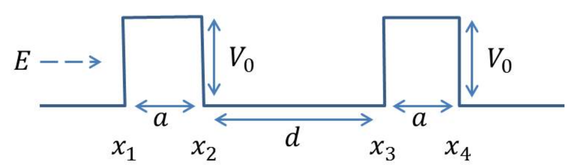

Figure 2: Square barrier model.

The problem is now simply a matter of transmission through zero, one, or two identical square barriers (see Figure 2), depending on the spin state. Although this is a standard textbook problem, it is useful for our purposes to provide a brief summary of the solution here. The state of an electron is a superposition of right- and left-moving plane waves, with different coefficients and wavevectors in the different potential regions. By simply enforcing the continuity conditions at the boundaries between regions, one straightforwardly obtains the transmission coefficient for a single barrier as $T_1 = \left\{1 + \frac{(k^2+\kappa^2)^2}{4k^2\kappa^2}\sinh^2 \kappa a\right\}^{-1}$, where $k \equiv \sqrt{2mE/\hbar^2}$ and $\kappa \equiv \sqrt{2m(V_0 - E)/\hbar^2}$. The transmission coefficient for a double barrier can likewise be shown to be $T_2 = T_1^2/\big|1 + R_1 \exp\big(-2i(\phi_1 + ka +$

$kd))\Big|^2$, where $R_1 = 1 - T_1$ and $\phi_1 = \arctan\left(\frac{k^2-\kappa^2}{2k\kappa}\tanh(\kappa a)\right) - ka$. In order to operate the system as a parity meter, we want to have $T_1 \ll 1$ and $T_2 = 1$ for the same values of V_0, a, etc. The first condition is to allow the current measurement to distinguish between the two parity cases of the spins. For the parameter values we have assumed above, this is satisfied for any $E < 600\ \mu eV$. The second condition ensures that the states within the even parity subspace are not distinguished by the measurement. This requires that $\arctan\left(\frac{k^2-\kappa^2}{2k\kappa}\tanh(\kappa a)\right) + kd = \frac{(2n+1)\pi}{2}, n = 0,1,2, \ldots$, which, e.g., for $E = 350\ \mu eV$, is satisfied for any of a set of inter-qubit distances $d \cong 50\ nm + n \times 125\ nm$.

Remarkably, this simple example of a two-qubit parity check can immediately be extended to an N-qubit case by choosing the inter-qubit distance such that the second condition above is satisfied for any two of the barriers, with all other barriers set to zero. This is satisfied so long as $k(d + a) = p\pi, p = 1,2,3, \ldots$ simultaneously with the conditions above, resulting in the condition $\frac{k^2-\kappa^2}{2k\kappa}\tanh(\kappa a) = -\cot(ka)$, which leads to $E \cong 350\ \mu eV$ and the inter-qubit distances quoted above. This ensures unit transmission for two "on" barriers separated by any number of "off" barriers. Since one can straightforwardly show that the transmission of any segmented scattering potential can be written in terms of the transmission of two subdivisions of that potential as $T_{tot} \propto \frac{T_1 T_2}{1-R_1 R_2 e^{i\gamma}}$, it follows that, under the special conditions derived above, one obtains unit transmission through any even number of barriers since one may recursively subdivide the system into resonant double-barrier structures with $T_1 = T_2 = 1$, $R_1 = R_2 = 0$. In the same way, it also follows that there is negligible transmission through any odd number of barriers, since one may subdivide the system into a string of double-barriers for which $T_1 = 1$ and $R_1 = 0$ and a single barrier for which $T_2 \ll 1$. Thus, one obtains an N-qubit parity check.

While arbitrarily large resonant inter-qubit distances exist in the solution above, this of course neglects the finite phase decoherence length of the transport electrons. However, this can be many microns, which is more than enough for practical purposes. Another consideration that arises at large distances is that the resonance peak narrows, requiring stricter stability in the parameter values in order to reliably observe the interference effect. Specifically, for the particular parameters quoted above, the transmission near a resonance has a Breit-Wigner lineshape with a characteristic width of about 25 neV at an interqubit distance of about a micron. Thus, the injected transport electrons need to have energies controlled with a similar precision in order to avoid distinguishing between states in the even-parity subspace. This becomes even more important as one includes more and more qubits.

Although everything has been couched in terms of singlet-triplet spin qubits due to their favorable coherence times, it is worth noting that everything developed in this proposal will also apply to charge qubits, hybrid qubits, triple-dot exchange-only or resonant exchange qubits, any other kind of qubit that can be temporarily charge-coupled to the conductance channel.

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