2010 F=ma Exam: Problem 23

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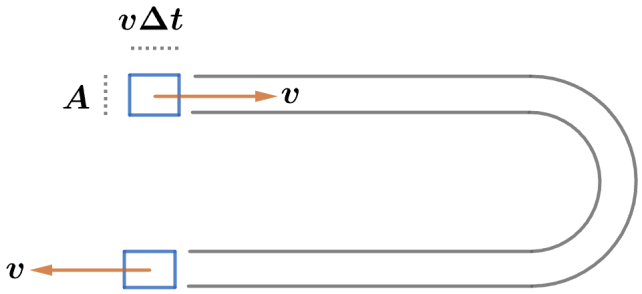
First, we study the force on one U-tube:



To find the force the stream of water exerts on the U-tube, we consider a small time interval Δt where a small mass Δm of water enters from the left at the top of the U-tube. Since the water is flowing at velocity v, the length of this segment is $\Delta L = v\Delta t$ so

$$\Delta m = \rho \Delta V = \rho A \Delta L = \rho A v \Delta t$$

At the same time, we have Δm of water leaving to the left with velocity $-v$ at the bottom of the U-tube. Thus, in time Δt, water of mass Δm effectively has its velocity changed from v to $-v$. This change in momentum can be attributed to the force from the U-tube on the water,

$$F = \frac{\Delta p}{\Delta t} = \frac{(\Delta m)v - (\Delta m)(-v)}{\Delta t} = 2\frac{(\Delta m)v}{\Delta t} = 2\rho A v^2$$

By Newton's 3rd law, this is also the force the water exerts on the U-tube. Going to the tube assembly of two U-tubes, since the net force is zero, we balance forces from each side:

$$F_1 = 2\rho A v^2 = 2\rho A' v'^2 = F_2$$

$$v' = v\sqrt{\frac{A}{A'}} = \sqrt{2}v$$

so the answer is $\boxed{\text{C}}$.