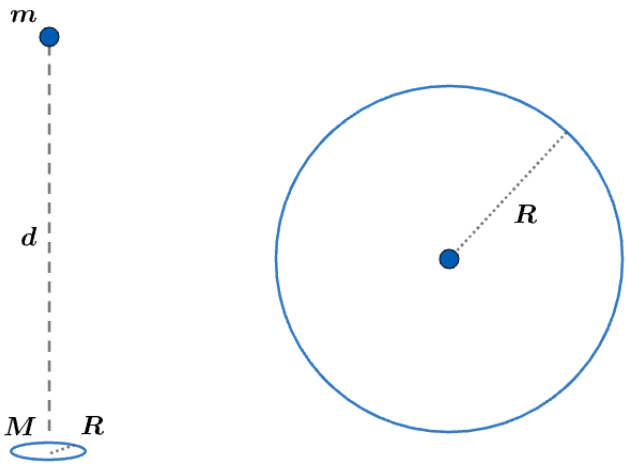
2008 F=ma Exam: Problem 18

Kevin S. Huang



Recall the gravitational potential energy between two particles of masses m and M separated by distance r is

$$U = -\frac{GMm}{r}$$

In our case, the particle starts very far away so there is no potential energy in the beginning,

$$U_i = 0$$

The particle maximizes its speed when it minimizes its potential energy. This occurs when the particle is closest to the ring i.e. located at the center. At this point, the distance between the particle and all parts of the ring is the radius R so the potential energy is

$$U_f = -\frac{GMm}{R}$$

where M is the mass of the ring. By conservation of energy,

$$U_i = K + U_f$$
$$0 = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

so we have

$$v = \sqrt{\frac{2GM}{R}} = \sqrt{4\pi G\lambda}$$

using $M = 2\pi R\lambda$ with λ being the linear mass density. Since the maximum speed is independent of radius, the answer is $\boxed{\text{C}}$.