2019B F=ma Exam: Problem 16

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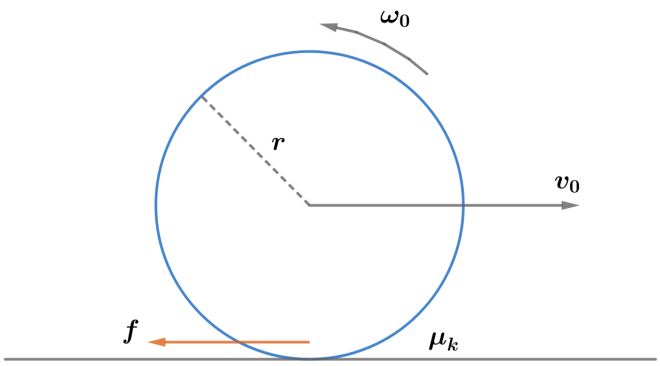

The hoop is initially slipping so we have kinetic friction,

$$f = Ma$$
$$fr = I\alpha$$

which means

$$a = \frac{f}{M} = \mu g$$
$$\alpha = \frac{fr}{I} = \frac{fr}{Mr^2} = \frac{\mu g}{r}$$

We find when the hoop starts rolling without slipping:

$$v(t) = r\omega(t)$$
$$(-v_0 + at) = r(\omega_0 - \alpha t)$$
$$-v_0 + at = r\left(\frac{3v_0}{r} - \alpha t\right)$$
$$-v_0 + \mu g t = 3v_0 - \mu g t$$
$$T = \frac{2v_0}{\mu g}$$

Note that

$$v(T) = -v_0 + a\left(\frac{2v_0}{\mu g}\right) = v_0$$

so by symmetry, this is also when the hoop returns to its starting position. Thus, the answer is $\boxed{\text{B}}$.