2019B F=ma Exam: Problem 11

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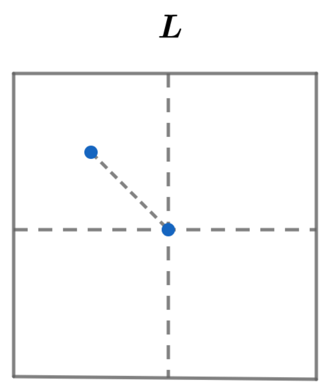
Recall the moment of inertia of a disk is

$$I = \frac{1}{2}MR^2$$

Let's find the moment of inertia of square I_{sq} about a perpendicular axis through its center:



We have

$$I_{sq} = 4I_{small,sq,cr}$$

By the parallel-axis theorem,

$$I_{small,sq,cr} = I_{small,sq} + \left(\frac{M}{4}\right)\left(\frac{L\sqrt{2}}{4}\right)^2$$

Since $I \propto ML^2$ and $M \propto L^2$, $I \propto L^4$:

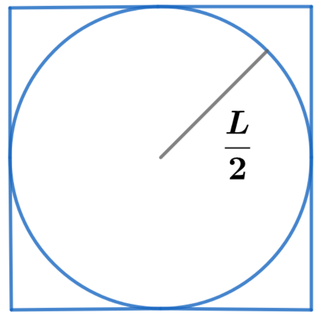
$$I_{small,sq} = \frac{I_{sq}}{16}$$

so

$$I_{sq} = 4\left[\frac{I_{sq}}{16} + \left(\frac{M}{4}\right)\left(\frac{L\sqrt{2}}{4}\right)^2\right]$$

$$\frac{3I_{sq}}{4} = \frac{ML^2}{8}$$

$$I_{sq} = \frac{1}{6}ML^2$$



The moment of inertia of the removed disk is

$$I_{disk} = \frac{1}{2}\left[\frac{\pi(L/2)^2}{L^2}M\right](L/2)^2 = \frac{\pi}{32}ML^2$$

Thus, the moment of inertia of the remaining shape is

$$I = I_{sq} - I_{disk} = \left(\frac{1}{6} - \frac{\pi}{32}\right)ML^2$$

so the answer is $\boxed{\text{A}}$.