2019A F=ma Exam: Problem 10

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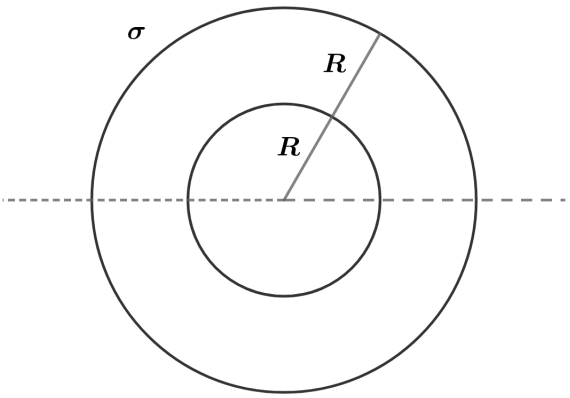

Recall the moment of inertia of a disk is given by

$$I = \frac{1}{2}MR^2$$

In terms of the mass density σ,

$$I = \frac{1}{2}(\sigma\pi R^2)R^2 = \frac{\pi}{2}\sigma R^4$$

The initial object has moment of inertia:

$$I_0 = \frac{\pi}{2}\sigma(2R)^4$$

After removing the smaller disk,

$$I_1 = \frac{\pi}{2}\sigma(2R)^4 - \frac{\pi}{2}\sigma(R)^4$$

Finally cutting in half,

$$I = \frac{1}{2}\left[\frac{\pi}{2}\sigma(2R)^4 - \frac{\pi}{2}\sigma(R)^4\right] = \frac{15\pi}{4}\sigma R^4$$

The mass of the remaining object is

$$M = \frac{1}{2}[\pi(2R)^2 - \pi(R)^2]\sigma = \frac{3\pi}{2}\sigma R^2$$

Hence,

$$I = \frac{5}{2}MR^2$$

so the answer is $\boxed{\text{D}}$.