2018A F=ma Exam: Problem 7

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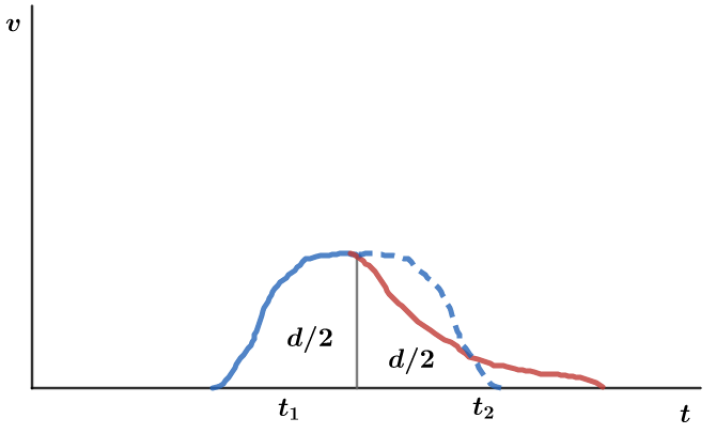

We have the symmetrical constraint $|a| \leq a_0$.

Consider the $v - t$ diagram of a possible path. For the path of shortest time, the time t_1 taken to traverse the first half (blue) of d must be the same as the time t_2 taken for the second half (red).

Reasoning: Suppose without loss of generality that $t_2 > t_1$. Then we can run the blue path in reverse for the second half to lower our total time (dotted curve). Thus, the original path was not the shortest.

Our problem is reduced to finding the path of shortest time to traverse $d/2$ which is simply constant acceleration at a_0. Thus,

$$\frac{1}{2}a_0 t_1^2 = \frac{d}{2}$$

$$t_1 = \sqrt{\frac{d}{a_0}}$$

From above,

$$t_2 = t_1 = \sqrt{\frac{d}{a_0}}$$

which corresponds to constant deceleration at $-a_0$ for the second half. Therefore,

$$t = t_1 + t_2 = 2\sqrt{\frac{d}{a_0}}$$

so the answer is $\boxed{\text{E}}$.