2022B F=ma Exam: Problem 19

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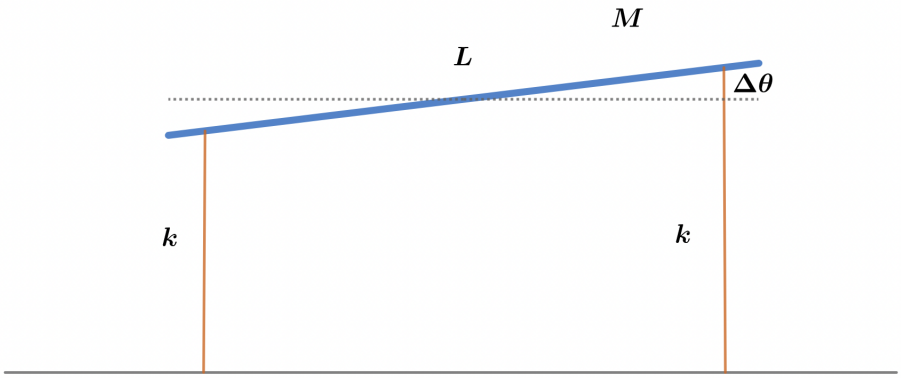

If the rod is rotated by a small angle $\Delta\theta$, the restoring torque is

$$\tau = -2F_{\text{spring}}\left(\frac{L}{2}\right) = -2\left(k\frac{L}{2}\Delta\theta\right)\frac{L}{2} = -\frac{kL^2\Delta\theta}{2}$$

Using the rotational form of Newton's 2nd law,

$$I\ddot{\theta} = -\frac{kL^2\theta}{2}$$

$$\ddot{\theta} = -\frac{kL^2}{2I}\theta$$

which corresponds to simple harmonic motion with angular frequency

$$\omega = \sqrt{\frac{kL^2}{2I}} = \sqrt{\frac{kL^2}{ML^2/6}} = \sqrt{\frac{6k}{M}}$$

since $I = ML^2/12$ for a rod. The frequency is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$$

so the answer is $\boxed{\text{E}}$.