2022A F=ma Exam: Problem 11

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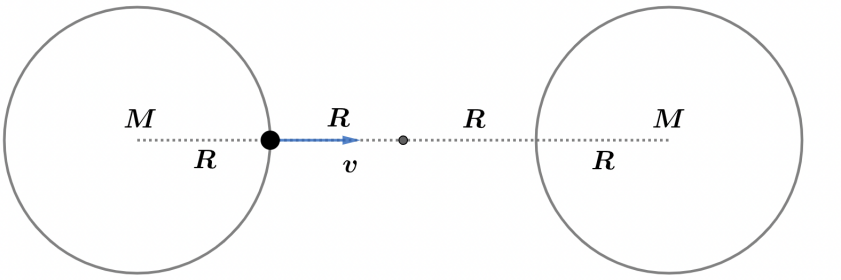

The gravitational force is zero in the middle of the two planets, so the rocket can just make it to the other planet by arriving at the middle with zero velocity. The initial energy is

$$K_i = \frac{1}{2}mv^2$$
$$U_i = -\frac{GMm}{R} - \frac{GMm}{3R} = -\frac{4GMm}{3R}$$
$$E_i = K_i + U_i = \frac{1}{2}mv^2 - \frac{4GMm}{3R}$$

The energy at the middle should be

$$K_f = 0$$
$$U_f = -\frac{GMm}{2R} - \frac{GMm}{2R} = -\frac{GMm}{R}$$
$$E_f = K_f + U_f = -\frac{GMm}{R}$$

By conservation of energy,

$$\frac{1}{2}mv^2 - \frac{4GMm}{3R} = -\frac{GMm}{R}$$
$$\frac{1}{2}v^2 = \frac{GM}{3R}$$
$$v = \sqrt{\frac{2GM}{3R}}$$

so the answer is $\boxed{\text{D}}$.