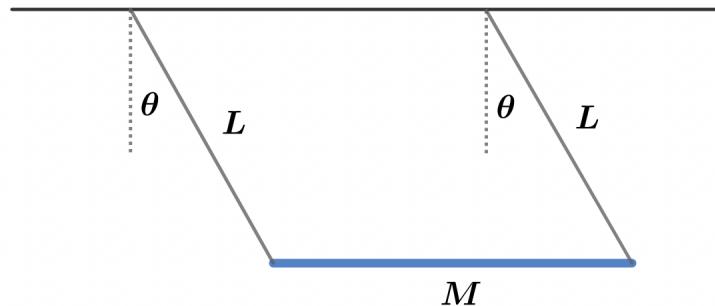


2022A F=ma Exam: Problem 10

Kevin S. Huang



Setting the ceiling as the potential energy reference, the potential energy of the system is

$$U = -MgL \cos \theta \approx -MgL \left(1 - \frac{\theta^2}{2}\right)$$

using the small angle approximation. The rod swings without changing its orientation so the kinetic energy is

$$K = \frac{1}{2}M{v_{CM}}^2 = \frac{1}{2}ML^2\dot{\theta}^2$$

The total energy is

$$E = K + U = \frac{1}{2}ML^2\dot{\theta}^2 - MgL \left(1 - \frac{\theta^2}{2}\right)$$

By conservation of energy,

$$\begin{aligned} \frac{dE}{dt} &= ML^2\dot{\theta}\ddot{\theta} + MgL\theta\dot{\theta} = 0 \\ L\ddot{\theta} + g\theta &= 0 \\ \ddot{\theta} &= -\frac{g}{L}\theta \end{aligned}$$

which corresponds to simple harmonic motion with angular frequency $\omega = \sqrt{g/L}$. The period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

so the answer is A.