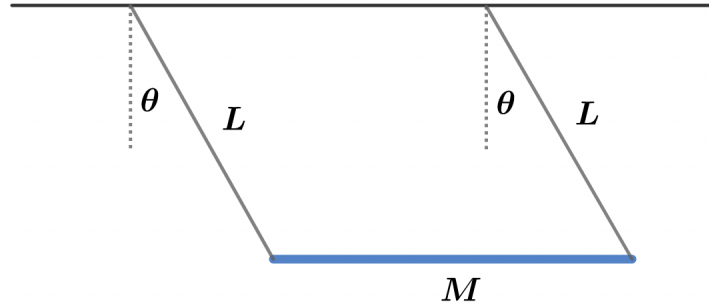


2022A F=ma Exam: Problem 10

Kevin S. Huang



Setting the ceiling as the potential energy reference, the potential energy of the system is

$$U = -MgL \cos \theta \approx -MgL \left(1 - \frac{\theta^2}{2}\right)$$

using the small angle approximation. The rod swings without changing its orientation so the kinetic energy is

$$K = \frac{1}{2} M v_{CM}^2 = \frac{1}{2} M L^2 \dot{\theta}^2$$

The total energy is

$$E = K + U = \frac{1}{2} M L^2 \dot{\theta}^2 - M g L \left(1 - \frac{\theta^2}{2}\right)$$

By conservation of energy,

$$\frac{dE}{dt} = M L^2 \dot{\theta} \ddot{\theta} + M g L \theta \dot{\theta} = 0$$

$$L \ddot{\theta} + g \theta = 0$$

$$\ddot{\theta} = -\frac{g}{L} \theta$$

which corresponds to simple harmonic motion with angular frequency $\omega = \sqrt{g/L}$. The period is

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}}$$

so the answer is \boxed{A} .