a) Hamiltonian $H = \frac{p^2}{2m} + V(\vec{r})$

commutation relations: $[x_i, p_j] = i\hbar\delta_{ij}$

$$[H, x_i] = \left[\frac{p^2}{2m} + V(\vec{r}), x_i\right] = \frac{1}{2m}[p^2, x_i] = \frac{1}{2m}[p_j p_j, x_i]$$

$$= \frac{1}{2m}\Big(p_j \underbrace{[p_j, x_i]}_{-i\hbar\delta_{ij}} + \underbrace{[p_j, x_i]}_{-i\hbar\delta_{ij}} p_j\Big) = -i\hbar\frac{p_i}{m}$$

$$\frac{d\langle A\rangle}{dt} = \left\langle \frac{\partial A}{\partial t}\right\rangle + \frac{i}{\hbar}\langle [H, A]\rangle$$

$$[H, \vec{r}] = \frac{\hbar}{i}\frac{\vec{p}}{m} \longrightarrow \boxed{\frac{d\langle\vec{r}\rangle_t}{dt} = \frac{\langle\vec{p}\rangle_t}{m}}$$

$$[H, \vec{p}] = \left[\frac{p^2}{2m} + V(\vec{r}), \vec{p}\right] = [V(\vec{r}), \vec{p}] = -\frac{\hbar}{i}\vec{\nabla}V(\vec{r})$$

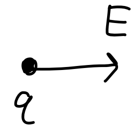
$$[V(\vec{r}), \vec{p}]\psi(\vec{r}) = [V(\vec{r}), \tfrac{\hbar}{i}\vec{\nabla}]\psi(\vec{r})$$
$$= V(\vec{r})\frac{\hbar}{i}\vec{\nabla}\psi(\vec{r}) - \frac{\hbar}{i}\vec{\nabla}(V(\vec{r})\psi(\vec{r}))$$
$$= V(\vec{r})\frac{\hbar}{i}\vec{\nabla}\psi(\vec{r}) - \frac{\hbar}{i}\psi(\vec{r})\vec{\nabla}V(\vec{r}) - \frac{\hbar}{i}V(\vec{r})\vec{\nabla}\psi(\vec{r})$$
$$= \left(-\frac{\hbar}{i}\vec{\nabla}V(\vec{r})\right)\psi(\vec{r})$$

$$\boxed{\frac{d\langle\vec{p}\rangle_t}{dt} = -\langle\nabla\vec{V}\rangle_t}$$

- classical equations of motion obeyed in expectation

- NOT expectation values obey classical equations

b) rename $x \to y$



$t<0$: $H = \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2 y^2$

$t>0$: $H = \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2 y^2 - qEy$

$$= \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2(y-y_0)^2 - \frac{1}{2}m\omega^2 y_0^2 \longrightarrow y_0 = \frac{qE}{m\omega^2}$$

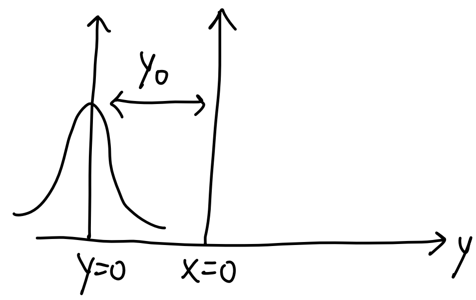
define $\left.\begin{aligned} x &= y - y_0 \\ p_x &= p_y \end{aligned}\right\}$ $\quad H = \dfrac{p_x^2}{2m} + \dfrac{1}{2} m\omega^2 x^2 + \varepsilon,$ $\quad \varepsilon = -\dfrac{1}{2} m\omega^2 y_0^2 = -\dfrac{1}{2}\dfrac{q^2E^2}{m\omega^2}$

Wavefunction at $t=0$:

$$\psi(x,0) = \psi_0(x+y_0)$$

$\uparrow$

example of a coherent state



$$a_y \psi(t=0) = 0$$

$$a_y = \frac{y\sqrt{\frac{m\omega}{\hbar}} + i p_y \frac{1}{\sqrt{m\hbar\omega}}}{\sqrt{2}} = \frac{(x+y_0)\sqrt{\frac{m\omega}{\hbar}} + i p_x \frac{1}{\sqrt{m\hbar\omega}}}{\sqrt{2}}$$

$$= a_x + y_0\sqrt{\frac{m\omega}{2\hbar}}$$

coherent state: $a\Psi_\alpha = \alpha\Psi_\alpha$

with $\alpha = -y_0\sqrt{\frac{m\omega}{2\hbar}}$

$$\rightarrow \left(a_x + y_0\sqrt{\frac{m\omega}{2\hbar}}\right)\psi(t=0) = 0 \rightarrow \overset{a_x}{\overset{\downarrow}{a}}\psi(t=0) = -y_0\sqrt{\frac{m\omega}{2\hbar}}\,\psi(t=0)$$

For a coherent state,

$$\Psi_\alpha(x) = e^{-i(\mathrm{Re}\,\alpha)(\mathrm{Im}\,\alpha)} e^{i\sqrt{2}(\mathrm{Im}\,\alpha)x} \chi_0\left(x - \sqrt{\frac{2\hbar}{m\omega}}\,\mathrm{Re}\,\alpha\right)$$

$$\psi(t) = e^{-iE_0 t/\hbar}\,\Psi_{\alpha e^{-i\omega t}}$$

in our case, $\alpha \in \mathbb{R}$

$$\boxed{\psi(t) = e^{-iE_0 t/\hbar} e^{i\alpha^2\cos(\omega t)\sin(\omega t)} e^{i\sqrt{2}(-\alpha\sin\omega t)\sqrt{\frac{m\omega}{\hbar}}x} \chi_0\left(x - \sqrt{\frac{2\hbar}{m\omega}}\,\alpha\cos(\omega t)\right)}$$