microscopic $\longrightarrow$ macroscopic
statistical mechanics

essence of statistical mechanics
large N limit
deviations are negligible (improbable)

Ω multiplicity

$g(\varepsilon)$ density of states

$S = k_B \ln g$ entropy

entropy will add

$$\frac{1}{T} = k_B\left(\frac{\partial S}{\partial U}\right)$$

$T_1 = T_2$

two systems in contact exchange energy to maximize entropy

"Laws":

0: $T_1 = T_2, T_2 = T_3 \rightarrow T_1 = T_3$

1: Heat is energy

2: S increases when you remove a constraint

3: $S \rightarrow$ finite value when $T \rightarrow 0$

$S(T=0) = k_B \ln[g(T=0)]$

$P(\varepsilon) \sim e^{-\beta\varepsilon}$; $\beta = \frac{1}{k_B T}$

$$P(\varepsilon) = \frac{e^{-\beta\varepsilon}}{\underbrace{\sum_{\text{states}} e^{-\beta\varepsilon}}_{Z \text{ partition function}}}$$

$$U = \langle E \rangle = \sum p(\varepsilon)\varepsilon$$
$$= \frac{\sum \varepsilon e^{-\beta\varepsilon}}{Z} = \frac{k_B T^2}{Z}\frac{\partial Z}{\partial T}$$
$$= -\frac{\partial \ln Z}{\partial \beta}$$

$$Z = \int_\varepsilon g(\varepsilon) e^{-\beta\varepsilon} d\varepsilon$$

chemical potential

$$dU = \underbrace{T\,dS}_{\text{heat}} - \underbrace{p\,dV}_{\text{work}} + \underbrace{\mu\,dN}_{\text{particle numbers}}$$

$T_1 = T_2$
$p_1 = p_2$
$\mu_1 = \mu_2$

Const. T : Helmholtz $F = U - TS$

const. p : Enthalpy $H = U + pV$

const. T, P : Gibbs $G = U - TS + pV$

$$\int_{-\infty}^{+\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$\ln N! \approx N \ln N$

Free energy:

Const temp. (Bath)

$$F = U - TS$$
$$= -k_B T \ln Z \rightarrow Z = e^{-\beta F}$$

$$dF = d(U - TS)$$
$$= T\,dS - p\,dV - \mu\,dN - T\,dS - S\,dT$$

$$dF = -S\,dT - p\,dV + \mu\,dN$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_{V,N}$$

$$P = -\left(\frac{\partial F}{\partial V}\right)_{T,N}$$