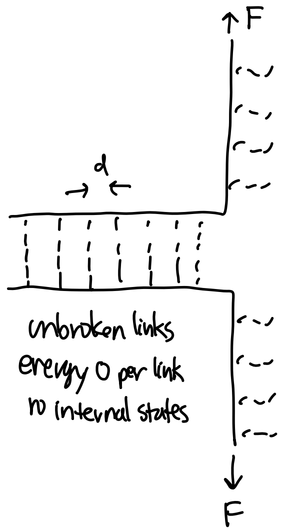


broken links

energy ε per broken link

g internal states for each half

a) $g=1$ mean number $\bar{n}$ of broken links

$k_BT \gg \varepsilon$

$F=0$

$\bar{n}d \ll L$

partition function $Z = \sum_{n=0}^{N} e^{-\beta n\varepsilon} \approx \sum_{n=0}^{\infty} e^{-\beta n \varepsilon}$

$= \dfrac{1}{1-e^{-\beta\varepsilon}}$

$Z = \sum e^{-\beta E}$

$\dfrac{\partial Z}{\partial \beta} = \sum -E\, e^{-\beta E}$

$\langle E \rangle = -\dfrac{1}{Z}\dfrac{\partial Z}{\partial \beta} = -\dfrac{\partial \log Z}{\partial \beta}$

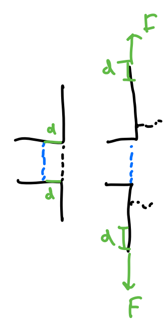
$\log Z = -\log(1-e^{-\beta\varepsilon})$

$-\dfrac{\partial}{\partial\beta}\log Z = \dfrac{1}{1-e^{-\beta\varepsilon}} \cdot -e^{-\beta\varepsilon} \cdot -\varepsilon = \dfrac{\varepsilon\, e^{-\beta\varepsilon}}{1-e^{-\beta\varepsilon}}$

$\rightarrow \quad \bar{n} = \dfrac{\langle E \rangle}{\varepsilon} = \dfrac{e^{-\beta\varepsilon}}{1-e^{-\beta\varepsilon}} = \dfrac{1}{e^{\beta\varepsilon}-1} \underset{k_BT \gg \varepsilon}{\approx} \dfrac{1}{1+\frac{\varepsilon}{k_BT}-1} = \dfrac{k_BT}{\varepsilon}$

$$\boxed{\bar{n} = \frac{k_BT}{\varepsilon}}$$

When a force is applied, the energy per link becomes $\varepsilon' = \varepsilon - 2Fd$



$$\bar{n} \rightarrow \frac{k_BT}{\varepsilon'} = \boxed{\frac{k_BT}{\varepsilon - 2Fd}}$$

b) $g > 1$

internal states

partition function $$Z = \sum_{n=0}^{N} (g^2)^n e^{-\beta n \varepsilon'} \approx \sum_{n=0}^{\infty} (g^2)^n e^{-\beta n \varepsilon'}$$

$$= \sum_{n=0}^{\infty} g^{2n} e^{-\beta n(\varepsilon - 2Fd)}$$

$$\boxed{Z = \frac{1}{1 - g^2 e^{-\beta(\varepsilon - 2Fd)}}}$$

free energy

$$G = -k_B T \log Z = \boxed{k_B T \log\left[1 - g^2 e^{-\beta(\varepsilon - 2Fd)}\right]}$$

At $T = T_c$, $\overset{Z\to\infty}{G \to -\infty}$ since there will be an "infinite" number of broken links

$$1 - g^2 e^{-\beta_c(\varepsilon - 2Fd)} = 0$$

$$e^{-\beta_c(\varepsilon - 2Fd)} = \frac{1}{g^2}$$

$$-\beta_c(\varepsilon - 2Fd) = -\log g^2$$

$$\frac{\varepsilon - 2Fd}{k_B T_c} = 2\log g \longrightarrow \boxed{T_c = \frac{\varepsilon - 2Fd}{2k_B \log g}}$$

c) $$Z = \frac{1}{1 - g^2 e^{-\beta(\varepsilon - 2Fd)}}$$

$$\bar{n} = \frac{\langle E \rangle}{\varepsilon'} = -\frac{1}{\varepsilon'}\frac{\partial \log Z}{\partial \beta} = \frac{1}{\varepsilon'}\frac{\partial}{\partial \beta}\log\left[1 - g^2 e^{-\beta(\varepsilon - 2Fd)}\right]$$

$$= \frac{1}{\varepsilon'}\frac{1}{1 - g^2 e^{-\beta(\varepsilon - 2Fd)}} \cdot -g^2 e^{-\beta(\varepsilon - 2Fd)} \cdot -(\varepsilon - 2Fd)$$

$$\bar{n} = \underbrace{\frac{\varepsilon - 2Fd}{\varepsilon'}}_{1} \frac{g^2 e^{-\beta(\varepsilon - 2Fd)}}{1 - g^2 e^{-\beta(\varepsilon - 2Fd)}}$$

$$\boxed{\bar{n} = \frac{1}{g^{-2} e^{\beta(\varepsilon - 2Fd)} - 1}}$$